MAT 371 HOMEWORK 4

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1. Let \( \{x_n\} \) be a sequence, and suppose \( x_n \to x \). Define

\[
t_n = \frac{1}{n} \sum_{k=1}^{n} x_k.
\]

Prove that \( t_n \to x \). Hint: break the sum \( \sum_{k=1}^{n} x_k \) into two pieces, one with a fixed number of terms, and the other with all terms close to \( x \).

2. (a) Prove that the sequence

\[
\left( \frac{n^2 + 3n + 5 \sin n}{3n^2 + 4} \right)
\]

has a convergent subsequence.

(b) Does the sequence \( (n(-1)^n) \) have a convergent subsequence? Why or why not?

3. Prove that for all \( x > 0 \), \( x^{1/n} \to 1 \). Hint: show that is suffices to consider the case \( x > 1 \). Then show that if \( \epsilon > 0 \) then \( x^{1/n} \) is eventually smaller than \( 1 + \epsilon \).

4. Let \( x_0 > 1 \), and for \( n \in \mathbb{N} \) define

\[
x_n = \frac{1 + x_{n-1}}{2}.
\]

Prove that \( x_n \to 1 \). Hint: show the sequence \( (x_n) \) is monotone, and then argue by contradiction that \( \inf x_n \) cannot be greater than \( 1 \).