MAT 371 HOMEWORK 12

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1. For each $n \in \mathbb{N}$ define $f_n : (0, 1]$ by $f_n(x) = x^{1/n}$. Prove:
   
   (a) $f_n \to 1$ uniformly on $[a, 1]$ for every $a \in (0, 1)$.
   
   (b) $f_n \not\to 1$ uniformly on $(0, 1]$.

2. Let $f, f_1, f_2, \ldots : A \to \mathbb{R}$ and $x_1, x_2, \ldots \in A$. Suppose $f_n \to f$ uniformly and $f(x_n) \to L$. Prove that $f_n(x_n) \to L$.

3. In each part below, determine whether the series converges. If it converges, determine whether it converges absolutely or conditionally.
   
   (a) 
   
   $$\sum_{n=1}^{\infty} \frac{\sin(2n + 1)}{n^2}$$
   
   (b) 
   
   $$\sum_{n=2}^{\infty} \frac{1}{n \log n}$$

Date: Due: Monday, April 26.