MAT 371 HOMEWORK 1

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1. Let \( f: A \to B \).
   (a) Prove that \( C \subseteq f^{-1}(f(C)) \) for all \( C \subseteq A \).
   (b) Prove that \( f \) is 1-1 if and only if \( C = f^{-1}(f(C)) \) for all \( C \subseteq A \).

2. Let \( \{B_i\}_{i \in I} \) be a family of sets.
   (a) Prove that for every set \( A \),
   \[
   A \cup \bigcap_{i \in I} B_i = \bigcap_{i \in I} (A \cup B_i).
   \]
   (b) Prove that
   \[
   \left( \bigcap_{i \in I} B_i \right)^c = \bigcup_{i \in I} B_i^c.
   \]

3. Let \( f: A \to B \), let \( \{A_i\}_{i \in I} \) be a family of subsets of \( A \), and let \( \{B_j\}_{j \in J} \) be a family of subsets of \( B \). Prove:
   (a) \[
   f\left( \bigcap_{i \in I} A_i \right) \subseteq \bigcap_{i \in I} f(A_i).
   \]
   (b) \[
   f^{-1}\left( \bigcap_{j \in J} B_j \right) = \bigcap_{j \in J} f^{-1}(B_j).
   \]

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