MAT 342 Linear Algebra

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Test 1

Time limit: 1 hour, 15 minutes. No notes, books, calculators, etc. Write your solutions on the blank pages provided — nothing on the test page(s) will be graded. Write on only one side of each page and leave reasonable margins. Print your name clearly in the upper right corner of each page, including the test page(s). Turn in your exam with the test page(s) on top. Write your final solutions clearly and in logical order — do not turn in scratch pages. You must follow all explicit instructions and show all your reasoning. This exam has 5 problems, and all problems have equal credit.

1. Suppose that $A$ is a $7 \times 4$ matrix such that $Ax = 0$ has only the trivial solution. Let $R$ be the reduced echelon form of $A$. In this problem, only the answers will be graded — no reasons needed.

(a) How many leading columns does $R$ have?

(b) How many rows of $R$ are 0?

(c) Find $R$.

(d) Is Col $A = \mathbb{R}^7$?

(e) Is Row $A = M_{1 \times 4}$?

(f) Is Col $A^t = \mathbb{R}^4$?

2. Suppose that $A$ is a matrix with reduced echelon form $R = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, and that $A \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix}$. Find all solutions of $Ax = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

3. Let $A = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$, and suppose that the reduced echelon form of $A$ is $R = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \end{pmatrix}$. Find scalars $s, t$ such that $\begin{pmatrix} c \\ f \end{pmatrix} = s \begin{pmatrix} a \\ d \end{pmatrix} + t \begin{pmatrix} b \\ e \end{pmatrix}$.
4. (a) Find the inverse of the matrix \( A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 2 \end{pmatrix} \), and 

(b) use it to solve the system \( Ax = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \).

Use our standard method to find the inverse, and use the requested method to solve the system!

5. Suppose 2 \( \times \) 2 matrices \( A \) and \( B \) have the same column space.

(a) Do \( A \) and \( B \) have the same null space?

(b) If \( A \) is invertible, must \( B \) also be invertible?

In each part, either give a reason if you answer “yes”, or give a counterexample, with explanation, if you answer “no”!