Problem:

Let
\[ \vec{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad \vec{y} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad \text{and} \quad \vec{z} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \]

and let \( W = \text{span}\{\vec{x}, \vec{y}, \vec{z}\} \). Note that \( \vec{x}, \vec{y}, \vec{z} \) are orthogonal.

(a) (4 points) Find a vector \( \vec{w} \) such that \( \{\vec{x}, \vec{y}, \vec{z}, \vec{w}\} \) is an orthogonal basis of \( \mathbb{R}^4 \).

(b) (2 points) Find the orthogonal projection of \( \vec{w} \) onto \( W \) (an expression in terms of the vectors \( \vec{x}, \vec{y}, \vec{x}, \vec{w} \) is good enough).