Problem: Define $T \in L(\mathbb{R}^2)$ by

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x - y \\ x - 2y \end{pmatrix}.$$ 

Let $E$ be the standard basis of $\mathbb{R}^2$, and let $F$ be the basis

$$F = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 5 \end{pmatrix} \right\}.$$

(a) Find the matrix $[T]_E$ representing $T$ relative to $E$.

(b) Find the matrix $[T]_F$ representing $T$ relative to $F$. You may use the Change of Basis Theorem for matrices representing linear operators, but you may also use any alternative valid method. Just be sure to show your work and make it clear what method you use.