1. In this problem, only the answers will be graded — no reasons needed.

(a) Define \( T \in L(\mathbb{R}^2) \) by \( T(x, y) = (2x - y, x + 3y) \). Find the matrix which represents \( T \) relative to the standard basis.

(b) Let \( T \) be a linear operator on a 2-dimensional vector space \( V \), let \( E \) be a basis of \( V \), and let \( \vec{x} \in V \). If \( [T]_E = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \) and \( [\vec{x}]_E = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \), find \( [T(\vec{x})]_E \).

(c) Let \( A \) be a \( 7 \times 5 \) matrix, and let \( W = \text{Col} \ A \). If \( \dim W^\perp = 2 \), do the rows of \( A \) span \( M_{1 \times 5} \)?

(d) Find the determinant of \( \begin{pmatrix} 2 & 1 & 8 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix} \).

(e) Let \( \vec{x} \) and \( \vec{y} \) be vectors in an inner product space \( V \). Suppose 
\[
\|\vec{x}\| = 3, \quad \|\vec{y}\| = 4, \quad \langle \vec{x}, \vec{y} \rangle = 5.
\]
Find \( \cos \theta \), where \( \theta \) is the angle between \( \vec{x} \) and \( \vec{y} \).

(f) If \( T \in L(\mathbb{R}^5, \mathbb{R}^4) \) has nullity 1, is \( T \) onto?

2. Define \( T \in L(P_3, P_2) \) by 
\[
T(a + bx + cx^2 + dx^3) = a - 2b + 2d + (c + d)x + (2a - 4b + 2c + 6d)x^2.
\]

(a) Find the matrix which represents \( T \) relative to the standard bases.

(b) Find the reduced echelon form of the matrix of part (a).

(c) Use the results of parts (a) and (b) to find a basis of the range of \( T \).
3. Consider the system \( A\vec{x} = \vec{b} \), where
\[
A = \begin{pmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}.
\]

(a) Show that the normal equations for the system \( A\vec{x} = \vec{b} \) are
\[
\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \vec{x} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}.
\]

(b) Find the least squares solution of the system \( A\vec{x} = \vec{b} \).

(c) Find the vector in the column space of \( A \) which is closest to the vector \( \vec{b} \).

4. Let \( A \) be a \( 3 \times 3 \) symmetric matrix (of real numbers), and suppose that:
   - the eigenvalues of \( A \) are \(-1\) and 5;
   - the eigenspace associated to the eigenvalue \( \lambda = -1 \) has a basis \( \left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \right\} \);
   - the eigenspace associated to the eigenvalue \( \lambda = 5 \) has a basis \( \left\{ \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\} \).

Find an orthogonal matrix \( P \) and a diagonal matrix \( D \) such that \( D = P^{-1}AP \).

5. Let
\[
A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & -2 & 2 \end{pmatrix}.
\]

(a) Find the eigenvalues of \( A \).

(b) For each eigenvalue of \( A \), find a basis for the associated eigenspace.