1. In this problem, only the answers will be graded — no reasons needed.

(a) Let \( A \) be a 7 \times 11 matrix. Assume that the solution space of the homogeneous system \( A\vec{x} = \vec{0} \) has dimension 5. What is the dimension of the row space of \( A \)?

(b) If \( T \in L(\mathbb{R}^3, \mathbb{R}^4) \), is it possible for \( T \) to be onto?

(c) Define \( T \in L(\mathbb{R}^3) \) by \( T(\vec{x}) = A\vec{x} \), where \( A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \). Is \( T \) 1-1?

(d) Let \( A \in M_{5\times8} \) and \( \vec{b} \in \mathbb{R}^5 \). If \( \text{rank} \ A = 3 \) and \( \text{rank} \ (A \quad \vec{b}) = 4 \), is the linear system \( A\vec{x} = \vec{b} \) consistent?

(e) If a matrix is 5 \times 9, is it possible for its columns to be independent?

(f) If a matrix is 5 \times 9, is it possible for its rows to be independent?

2. Let

\[
A = \begin{pmatrix} 0 & -1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 0 & 1 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -2 & -2 \end{pmatrix}.
\]

(a) Show that the reduced echelon form of \( A \) is

\[
\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.
\]

(b) Use the result of part (a) to find (with explanation) the rank and nullity of \( A \).
3. In this problem, only the answers will be graded — no reasons needed.

(a) Given that the reduced echelon form of
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
5 & -1 & -1 & 0 & 1 & 0 \\
3 & -2 & -1 & 0 & 0 & 1
\end{pmatrix}
\]
is
\[
\begin{pmatrix}
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & -2 & 1 & -1 \\
0 & 0 & 1 & 7 & -2 & 1
\end{pmatrix},
\]
find the inverse of
\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
5 & -1 & -1 & 0 \\
3 & -2 & -1 & 0
\end{pmatrix}.
\]

(b) Given that the reduced echelon form of
\[
\begin{pmatrix}
0 & 1 & 0 & 1 \\
2 & 0 & -4 & -2 \\
0 & -3 & 1 & 1
\end{pmatrix}
\]
is
\[
\begin{pmatrix}
1 & 0 & 0 & 7 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 4
\end{pmatrix},
\]
express \( \begin{pmatrix} -2 \\ 1 \end{pmatrix} \) as a linear combination of
\[
\begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}.
\]

4. Let \( E \) be the standard basis of \( \mathbb{R}^2 \), and let \( F \) be the basis \( \{ (1, 2), (2, 6) \} \).

(a) Find the change of basis matrix \( P \) from \( F \) to \( E \).

(b) Find the change of basis matrix \( Q \) from \( E \) to \( F \).
5. (a) Let

\[ A = \begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}. \]

Given that the reduced echelon form of

\[ \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \]

is

\[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \]

are the vectors \( A, B, C \) independent in the vector space \( M_2 \)?

Be sure to give a brief indication of what the first matrix has to do with \( A, B, C \), and how the reduced echelon form gives you the answer.

(b) Let

\[ p(x) = 1 + 2x + 3x^2, \quad q(x) = 1 + x^2, \quad \text{and} \quad r(x) = -1 + 4x + 3x^2. \]

Given that the reduced echelon form of

\[ \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 4 \\ 3 & 1 & 3 \end{pmatrix} \]

is

\[ \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{pmatrix}, \]

do \( p, q, r \) span \( P_2 \)?

Be sure to give a brief indication of what the first matrix has to do with \( p, q, r \), and how the reduced echelon form gives you the answer.