Officially, the term “set” is undefined; any axiomatic mathematical theory must begin with some undefined terms, such as “point” and “line” in Euclidean geometry. Informally, however, we regard a set as a collection of objects. If $V$ is a set and $x$ is an object, officially (again) the term “element” in the phrase “$x$ is an element of $V$” is undefined; in axiomatic set theory only the axioms governing this relation would be specified. However, again informally, if we regard a set $V$ as a collection of objects, then we say $x$ is an element of $V$, and we write $x \in V$, to mean that $x$ is one of the objects in the collection $V$. The negation of $x \in V$, i.e., “$x$ is not an element of $V$”, is written $x \notin V$.

The main thing is that two sets $V$ and $W$ are equal exactly when they have the same elements, i.e., $V = W$ if and only if for all $x$ we have $x \in V$ if and only if $x \in W$.

Let $V$ and $W$ be sets. We say $W$ is a subset of $V$, written $W \subset V$, if for all $x \in W$ we have $x \in V$, equivalently for all $x$, if $x \in W$ then $x \in V$.

The empty set is the unique set $\emptyset$ with no elements. Thus, $\emptyset$ is characterized by the following property: for all $x$ we have $x \notin \emptyset$.

A set is finite if it has $n$ elements for some nonnegative integer $n$, otherwise it is infinite.

There is no set $V$ such that for all $x$ we have $x \in V$; such a set would lead to logical contradictions, the most famous being Russel’s Paradox.

A set can be specified in several ways:

1. in words, e.g., the set of real numbers greater than 2;
2. by listing the elements, e.g., $\{1, 2, 3\}$;
3. by listing a few elements and indicating that a pattern is to be continued, e.g., $\{1, 2, 3, \ldots, n\}$ (a finite set) or $\{1, 2, 3, \ldots\}$ (an infinite set);
4. by giving a defining property for membership, e.g., $\{x \in \mathbb{R} : x > 3\}$.

Let $V$ and $W$ be sets.

1. The intersection of $V$ and $W$ is the set $V \cap W := \{x : x \in V \text{ and } x \in W\}$.
2. The union of $V$ and $W$ is the set $V \cup W := \{x : x \in V \text{ or } x \in W\}$.
3. The difference of $V$ and $W$ is the set $V \setminus W := \{x \in V : x \notin W\}$.
4. $V$ and $W$ are disjoint if $V \cap W = \emptyset$. 