Exercise 1. In each part, determine whether the matrix is orthogonal:

(a) \[ A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \]

(b) \[ A = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ -2 & -1 \end{pmatrix} \]

(c) \[ A = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \]

Exercise 2. Let \{u_1, u_2, u_3\} be an orthonormal basis of an inner product space \(V\). Let
\[ x = u_1 + 3u_2 - 4u_3 \quad \text{and} \quad y = u_2 - 2u_3. \]

Find:

(a) \( \langle x, y \rangle \)

(b) \( \|x\| \)

(c) \( \|x - y\| \)

Exercise 3. Let \(V\) and \(W\) be finite-dimensional inner product spaces and \(T \in L(V,W)\). Prove that \(T^*T = I\) if and only if for every orthonormal basis \(E\) of \(V\) the image \(T(E)\) is orthonormal.

Exercise 4. Let \(A \in M_{m \times n}\). Prove that \(A^tA = I\) if and only if the columns of \(A\) are orthonormal.

Exercise 5. Let \(V\) be a finite-dimensional inner product space and \(T \in L(V)\). Prove that \(T\) is orthogonal if and only if for all \(x \in V\) we have
\[ \|T(x)\| = \|x\|. \]

Exercise 6. Let \(\theta \in \mathbb{R}\), and let \(T \in L(\mathbb{R}^2)\) be rotation by \(\theta\). Prove that \(T\) is orthogonal.

Exercise 7. Let \(T \in L(\mathbb{R}^2)\) be reflection across the 1st coordinate axis. Prove that \(T\) is orthogonal.
Exercise 8. Let $\lambda$ be an eigenvalue of an orthogonal operator $T$ on a finite-dimensional inner product space $V$. Prove that $|\lambda| = 1$. 