Exercise 1. Write
\[
\begin{pmatrix}
3 \\
0 \\
-1 \\
-4
\end{pmatrix}
\]
as a linear combination of the vectors in the standard basis of \(\mathbb{R}^4\).

Exercise 2. Write
\[
\begin{pmatrix}
2 & 1 & 0 \\
3 & -2 & 4
\end{pmatrix}
\]
as a linear combination of the vectors in the standard basis of \(M_{2\times3}\).

Exercise 3. Write
\[3 + 2x - 4x^2\]
as a linear combination of the vectors in the standard basis of the vector space of polynomials of degree at most 2.

Exercise 4. In each part, give a very short reason why the statement is true:

(a) The vectors
\[
\begin{pmatrix}
3 \\
1 \\
2
\end{pmatrix}, \begin{pmatrix}
1 \\
2 \\
5
\end{pmatrix}, \begin{pmatrix}
-1 \\
1 \\
3
\end{pmatrix}, \begin{pmatrix}
3 \\
2 \\
1
\end{pmatrix}
\]
are dependent.

(b) The vectors
\[
\begin{pmatrix}
1 & 0 \\
2 & 2
\end{pmatrix}, \begin{pmatrix}
1 & 1 \\
0 & 3
\end{pmatrix}, \begin{pmatrix}
1 & 1 \\
1 & 1
\end{pmatrix}
\]
do not span \(M_2\).

(c) The vectors
\[
\begin{pmatrix}
1 \\
1
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
2 \\
5
\end{pmatrix}
\]
form a basis of \(\mathbb{R}^2\).

Exercise 5. What is the dimension of the vector space of diagonal \(n \times n\) matrices? How about the lower triangular \(n \times n\) matrices?

Give reasons.
Exercise 6. Find a basis of \( \mathbb{R}^3 \) containing the vectors
\[
\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}.
\]
Be sure to justify your assertions.

Exercise 7. In the vector space of polynomials, let
\[ E = \{1 + x, 2x + x^2, 1 - 3x - 2x^2, 1 + x + x^2\}. \]
(a) Find a subset of \( E \) which is a basis for \( \text{span} \ E \).
(b) What is \( \text{dim}(\text{span} \ E) \)?
Be sure to give reasons.

Exercise 8. Let \( V \) be a finite-dimensional vector space, let \( W \) and \( Z \) be subspaces of \( V \), and let \( E \) be a basis of \( W \) and \( F \) a basis of \( Z \). Assume that \( W \cap Z = \{0\} \). Prove that \( E \cup F \) is independent.

Exercise 9. Let the set \( \mathbb{C} \) of complex numbers be regarded as a vector space (with real numbers as scalars). Find a basis of \( \mathbb{C} \).

Exercise 10.
(a) How many subspaces of \( \mathbb{R}^3 \) are there of dimension 0?
(b) How many subspaces of \( \mathbb{R}^3 \) are there of dimension 3?
(c) How many subspaces of \( \mathbb{R}^3 \) are there of dimension 1?
(d) How many subspaces of \( \mathbb{R}^3 \) are there of dimension 2?

Exercise 11.
(a) How many bases does the vector space \( \{0\} \) have? How many vectors are in each basis?
(b) How many bases does the vector space \( \mathbb{R} \) have? How many vectors are in each basis?