Exercise 1. Prove that \[
\begin{pmatrix}
1 \\
2
\end{pmatrix}
\quad \text{and} \quad 
\begin{pmatrix}
3 \\
-1
\end{pmatrix}
\]
are independent.

Exercise 2. Prove that 
\[2 - x + 3x^3, \quad 1 + x - x^2, \quad \text{and} \quad -1 + x\]
are independent.

Exercise 3. Decide whether 
\[
\begin{pmatrix}
1 & 1 \\
2 & 1
\end{pmatrix}, \quad 
\begin{pmatrix}
1 & 0 \\
1 & 1
\end{pmatrix}, \quad \text{and} \quad 
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]
are independent.

Exercise 4. Let \( V \) be the vector space of all functions from \( \mathbb{R} \to \mathbb{R} \). Prove that the vectors \( e^x \) and \( e^{3x} \) in \( V \) are independent.

Exercise 5. Let 
\[
x = \begin{pmatrix}
1 \\
2 \\
0 \\
1
\end{pmatrix} \quad \text{and} \quad 
y = \begin{pmatrix}
0 \\
1 \\
1 \\
2
\end{pmatrix}.
\]
Find an example of \( z \in \mathbb{R}^4 \) such that \( \{x, y, z\} \) is independent, and prove your answer.

Exercise 6. Let \( V \) be a vector space and let \( x_1, \ldots, x_n \in V \). Assume that \( x_1, \ldots, x_n \) are independent. Prove that 
\[
x_1, \\
x_1 + x_2, \\
x_1 + x_2 + x_3, \\
\ldots \\
x_1 + x_2 + x_3 + \cdots + x_n
\]
are also independent.

Exercise 7. Prove that 
\[
\begin{pmatrix}
a \\
b
\end{pmatrix} \quad \text{and} \quad 
\begin{pmatrix}
c \\
d
\end{pmatrix}
\]
are dependent if and only if 
\[ad = bc.\]