Exercise 1. Let 
\[ W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x = y \right\}. \]
Prove that \( W \) is a subspace of \( \mathbb{R}^3 \).

Exercise 2. Let 
\[ W = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : x^2 = y^2 \right\}. \]
Is \( W \) a subspace of \( \mathbb{R}^2 \)? Prove your answer.

Exercise 3. Prove that the set of diagonal \( n \times n \) matrices is a subspace of \( M_n \).

Exercise 4. Let \( V \) be the vector space of all functions from \( \mathbb{R} \to \mathbb{R} \). In each part, determine whether the set \( W \) is a subspace of \( V \), and prove your answer:

(a) 
\[ W = \{ f \in V : f(1) = 2 \}. \]
(b) 
\[ W = \{ f \in V : f(1) = 2f(0) \}. \]
(c) 
\[ W = \{ f \in V : f(0) = 0 \text{ and } f(1) = 0 \}. \]
(d) 
\[ W = \{ f \in V : f(0) = 0 \text{ or } f(1) = 0 \}. \]

Exercise 5. Let \( V \) be the vector space of polynomials of degree at most 2, and let \( W \) be the set of polynomials of degree 2. Prove that \( W \) is not a subspace of \( V \).

Exercise 6. Let 
\[ W = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 : x \neq y \right\}. \]
Prove that \( W \) is not a subspace of \( \mathbb{R}^2 \).

Exercise 7. Let \( W \) and \( Z \) be subspaces of a vector space \( V \). Prove that \( W \cap Z \) is a subspace of \( V \).

Exercise 8. Let \( W \) and \( Z \) be subspaces of a vector space \( V \). Prove that 
\[ W + Z := \{ x + y : x \in W \text{ and } y \in Z \} \]
is a subspace of \( V \).