MAT 300 Notes on real numbers, Fall 2006

**Algebraic properties**

Here are the algebraic properties of real numbers that may be used in the homework for Section 2.3:

1. the axioms A1–A15 in Chapter 0;
2. the theorems and corollaries *which were proved* in Section 2.3.1;
3. uniqueness of the additive identity 0;
4. uniqueness of the multiplicative identity 1;
5. for each \( a \in \mathbb{R} \), uniqueness of the additive inverse (i.e., the “negative”) \(-a\);
6. for each \( a \in \mathbb{R} \setminus \{0\} \), uniqueness of the multiplicative inverse (i.e., the “reciprocal”) \(a^{-1}\);
7. special case of cancellation for addition: for all \( a, b \in \mathbb{R} \), if \( a + b = a \) then \( b = 0 \);
8. special case of cancellation for multiplication: for all \( a, b \in \mathbb{R} \), if \( a \neq 0 \) and \( ab = a \), then \( b = 1 \);
9. the definitions of subtraction and division;
10. extra properties proved in class: for all \( a, b \in \mathbb{R} \),
    (a) \( a = -b \) if and only if \(-a = b\);
    (b) \(-(a - b) = b - a\);
    (c) \(-(a + b) = -a - b\);
11. in each part of a problem, you may use any *preceding* part of that problem (and this rule also applies elsewhere).

**Order properties**

Here are the order properties of real numbers that may be used in the homework for Section 2.3:

1. the axioms A16–A19 in Chapter 0 (the only place A19 is used is in Exercise # 10f, and the remaining axioms A20–A22 are not used in Section 2.3);
2. the definitions of \( a > b \), \( a < b \), \( a \geq b \), and \( a \leq b \);
3. the theorems and corollaries *which were proved* in Section 2.3.2;
4. any algebraic property listed above;
5. embellishment of Exercise 9a: for all \( a, b, c \in \mathbb{R} \),
    (a) \( a < b \) if and only if \( a + c < b + c \);
    (b) \( a > b \) if and only if \( a + c > b + c \);
    (c) \( a \leq b \) if and only if \( a + c \leq b + c \);
    (d) \( a \geq b \) if and only if \( a + c \geq b + c \);
6. embellishment of definition of \( a > b \): for all \( a, b, c \in \mathbb{R} \),
    (a) \( a < b \) if and only if \( a - b < 0 \);
    (b) \( a \geq b \) if and only if \( a - b \geq 0 \);
    (c) \( a \leq b \) if and only if \( a - b \leq 0 \);
7. embellishment of Theorem 2.3.14: for all \( a, b \in \mathbb{R} \),
   (a) \( a > b \) if and only if \(-a < -b\);
   (b) \( a \geq b \) if and only if \(-a \leq -b\);

**Absolute value**

Here are the properties of absolute value that may be used in the homework for Section 2.3:

1. any property mentioned above;
2. the definition of absolute value;
3. the theorems and corollaries *which were proved* in Section 2.3.3;
4. embellishment of second part of definition of \(|x|\): for all \( x \in \mathbb{R} \), \(|x| = -x\) if and only if \( x \leq 0\);
5. *(after doing Exercise 9bc)* for all \( x, y, z \in \mathbb{R} \),
   (a) if \( x \leq y \) and \( y < z \) then \( x < z \);
   (b) if \( x, y \) and \( y \leq z \) then \( x < z \);
   (c) if \( x \geq y \) and \( y > z \) then \( x > z \);
   (d) if \( x > y \) and \( y \geq z \) then \( x > z \);