Name:

- You may use any part of a problem (solved or not) in the solution of any other problem, and in any later part of the same problem.
- You may ignore any hints, but you must follow all explicit instructions.
- Write each solution in the space provided, not on scratch paper.
- If you need more room, write on the back of the page. If you still need more room, ask for scratch paper. Do not use your own scratch paper.
- Be sure to give reasons for your answers (except where explicitly told otherwise)!
- Your solutions must be complete and organized, otherwise points may be deducted.
- There are 6 problems, and all problems have equal credit.
1. Negate the following statement, moving the negation inside as far as possible, and express the result in English (the textbook would say re-express the negation using equivalent positive statements):

   For every integer $n$, $n$ is even or $n \neq 0$. 

   The negation of this statement is:

   There exists an integer $n$ such that $n$ is odd and $n = 0$. 

   In English:

   There exists an integer $n$ such that $n$ is odd. 

   This means that not all integers are even, or that there is at least one integer that is odd and equal to zero.
2. In this problem, only the answer will be graded; you do not need to show any work!
Write the following statement in symbolic form (the textbook would say analyze the logical form):
For all $x \in \mathbb{R}$ there exists $n \in \mathbb{N}$ such that $n > x$. 

Solution:
\[ \forall x \in \mathbb{R} \exists n \in \mathbb{N} \ (n > x). \]
3. In this problem, only the answer will be graded; you do not need to show any work!

Write the following statement in English:

\((\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(x + y = 0)\).
4. Use a truth table to determine whether the following two statements are equivalent:

\[(P \lor Q) \rightarrow R \quad \text{and} \quad (P \rightarrow R) \lor (Q \rightarrow R)\]
5. Let $A$, $B$, and $C$ be sets. Verify the identity

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

by showing that for any object $x$, the statement that $x$ is an element of the left-hand set is equivalent to the statement that $x$ is an element of the right-hand set.

Hint: Start with what it means for an object $x$ to be an element of the left-hand set, then, using logical symbols, logical equivalences, and set-theoretic definitions, eventually end with what it means for $x$ to be an element of the right-hand set.
6. In this problem, only the answer will be graded; you do **not** need to show any work!

Let $I = \{1, 3, 5\}$, and for each $i \in I$ let $A_i = \{i, i + 1\}$. Find each of the following sets:

(a) $A_1$

(b) $A_3$

(c) $A_5$

(d) $\bigcup_{i \in I} A_i$

(e) $A_1 \cap A_3$

(f) $\bigcap_{i \in I} A_i$