MAT 300 Notes on Section 4.2
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Note: we will not cover composition of relations in this chapter!

A relation from $A$ to $B$ is a subset of $A \times B$.
If $R$ is a relation from $A$ to $B$, then:

The domain of $R$ is

$$\text{Dom}(R) = \{ x \in A \mid \text{there exists } y \in B \text{ such that } (x, y) \in R \}.$$  

The range of $R$ is

$$\text{Ran}(R) = \{ y \in B \mid \text{there exists } x \in A \text{ such that } (x, y) \in R \}.$$  

Then inverse of $R$ is

$$R^{-1} = \{ (y, x) \in B \times A \mid (x, y) \in R \}.$$  

Note $R^{-1}$ is a relation from $B$ to $A$.

Actually, a relation is just a set of ordered pairs — we don’t always care what the $A$ and $B$ are. Thus, if $R$ is a relation, then for all $x$ we have:

- $x \in \text{Dom}(R)$ if and only if there exists $y$ such that $(x, y) \in R$;
- $x \in \text{Ran}(R)$ if and only if there exists $y$ such that $(y, x) \in R$.

Also, for every ordered pair $(x, y)$, it makes sense to ask whether $(x, y) \in R$.  