Name:

- No books, notes or calculators of any kind are permitted on the test.
- Do not use your own scratch paper.
- Write each solution in the space provided, not on scratch paper.
- If you need more room, write on the back of the page. If you still need more room, ask for scratch paper.
- Be sure to give reasons for your answers (except where explicitly told otherwise)!
- Your solutions must be complete and organized, otherwise points may be deducted.
- There are 6 problems, and all problems have equal credit.
1. The parts of this problem are independent.
In this problem, only the answer will be graded; you do not need to show any work!

(a) True or False: If $D$ is the region in the $xy$-plane bounded by $y = x^2$ and $y = x + 2$, then evaluating $\int\int_D f(x, y) \, dA$ by integrating with respect to $x$ first would require 2 iterated integrals.

Solution: True

(b) True or False: The $x$-coordinate of the center of mass of a thin plate (“lamina”) occupying a region $D$ in the $xy$-plane is $\frac{1}{m} \int\int_D y \rho(x, y) \, dA$, where $m$ is the mass and $\rho$ is the density.

Solution: False

(c) If a double integral over the region $D$ indicated in the diagram is evaluated using polar coordinates, fill in the missing limits of integration:

$$\int \int \sin \theta \cdots \, dr \, d\theta$$

Solution: $\int_0^\pi \int_0^{\sin \theta} \cdots \, dr \, d\theta$

(d) Identify the type of surface whose equation in cylindrical coordinates is $z = r^2$

Solution: paraboloid

reason: $z = x^2 + y^2$

(e) Identify the type of surface whose equation in spherical coordinates is $\rho = 4 \sin \phi$.

Solution: torus

reason: $\rho^2 = 4 \rho \sin \phi$, so $r^2 + z^2 = 4r$

(f) If a triple integral over the hemisphere $E$ indicated in the diagram is evaluated using spherical coordinates, fill in the missing limit of integration:

$$\int_0^{2\pi} \int_{\pi/2}^\pi \int_0^2 \cdots \, d\rho \, d\phi \, d\theta$$

Solution: $\int_0^{2\pi} \int_{\pi/2}^\pi \int_0^2 \cdots \, d\rho \, d\phi \, d\theta$
2. The parts of this problem are independent.

(a) Reverse the order of integration:

\[ \int_0^1 \int_{y^2}^y f(x, y) \, dx \, dy \]

Solution:

\[ \int_0^1 \int_x^{\sqrt{x}} f(x, y) \, dy \, dx \]

(b) Set up, but do not evaluate, a double integral for the volume under the surface \( z = e^{xy} \) and above the triangular region in the \( xy \)-plane with vertices \((0, 0), (0, 2), \) and \((1, 1)\).

Solution:

\[ \int_0^1 \int_x^{2-x} e^{xy} \, dy \, dx \]
3. Evaluate the given integral by changing to polar coordinates:

\[
\int_0^2 \int_x^{\sqrt{4-x^2}} 3y \, dy \, dx
\]

Solution:

\[
\int_{\pi/4}^{\pi/2} \int_0^2 (3r \sin \theta) \, r \, dr \, d\theta = \int_{\pi/4}^{\pi/2} \int_0^2 3r^2 \sin \theta \, dr \, d\theta
\]

\[
= \int_0^2 3r^2 \, dr \int_{\pi/4}^{\pi/2} \sin \theta \, d\theta
\]

\[
= r^3 \bigg|_0^2 \left(- \cos \theta \right) \bigg|_{\pi/4}^{\pi/2}
\]

\[
= \frac{8}{\sqrt{2}}
\]
4. Evaluate the iterated integral
\[ \int_0^1 \int_1^y \int_0^z 2y \, dx \, dz \, dy \]

Solution:
\[
\int_0^1 \int_1^y \left[ 2yx \right]_{x=0}^{x=z} \, dz \, dy = \int_0^1 \int_1^y 2yz \, dz \, dy
\]
\[
= \int_0^1 \left[ yz^2 \right]_{z=0}^{z=y} \, dy
\]
\[
= \int_0^1 (y^3 - y) \, dy
\]
\[
= \left[ \frac{y^4}{4} - \frac{y^2}{2} \right]_0^1
\]
\[
= \frac{1}{4} - \frac{1}{2}
\]
\[
= -\frac{1}{4}
\]
5. Set up, but **do not evaluate**, an iterated integral for \( \iiint_E e^x \, dV \), where \( E \) is the region in the 1st octant bounded by the parabolic cylinder \( z = x^2 \), the plane \( z = x \), and the plane \( y = 2 \).

**Solution:**

\[
\int_0^2 \int_0^1 \int_{x^2}^x e^x \, dz \, dx \, dy
\]

The region \( E \):

![3D diagram of the region E](image)

Projection in \( xy \)-plane:

![xy-plane projection](image)
6. Convert the following integral from cylindrical coordinates to spherical coordinates. Write the resulting iterated integral as simply as you can, but **do not evaluate**!

\[ \int_0^{\pi/2} \int_0^1 \int_r^1 r^2 \cos \theta \, dz \, dr \, d\theta \]

**Solution:**

\[
\int_0^{\pi/2} \int_0^1 \int_r^1 r^2 \cos \theta \, dz \, dr \, d\theta = \int_0^{\pi/2} \int_0^1 \int_r^1 (r \cos \theta) \, r \, dz \, dr \, d\theta \\
= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec \phi} (\rho \sin \phi \cos \theta) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \\
= \int_0^{\pi/2} \int_0^{\pi/4} \int_0^{\sec \phi} \rho^3 \sin^2 \phi \cos \theta \, d\rho \, d\phi \, d\theta
\]

Region in cylindrical coordinates:

\[
\begin{aligned}
z &= 1 \\
z &= r
\end{aligned}
\]

Region in spherical coordinates:

\[
\begin{aligned}
\rho \cos \phi &= 1 \\
\phi &= \pi/4
\end{aligned}
\]