Name:

- Be sure to show all your work!

- No books, notes or calculators of any kind are permitted on the test.

- Do not use your own scratch paper.

- Write each solution in the space provided, not on scratch paper.

- If you need more room, write on the back of the page. If you still need more room, ask for scratch paper.

- Be sure to give reasons for your answers (except where explicitly told otherwise)!

- Points will probably be taken off if you do not give a clear indication of the method and write your solution neatly, in order, and clearly indicate your final answer.

- Points may be taken off for a correct answer with a mistake in the supporting work.
1. In this problem, only the answer will be graded; you do **not** need to show any work!

(a) What are the coordinates of the point in \( \mathbb{R}^3 \) (3-dimensional space) where the line \( x = 1 + t, y = 2 - 3t, z = 4t \) intersects the \( yz \)-plane?

(b) Find the radius of the sphere \( x^2 - 4x + y^2 + z^2 = 5 \).

(c) Compute \( 2a - b \) if \( a = 2i + j \) and \( b = -i + 3j \).

(d) If \( a \) is a unit vector, find the length of \( -3a \).

(e) Find a tangent vector to the curve \( r = ti + 2j + t^3k \) at the point \((1, 2, 1)\).

(f) Are the vectors \( \langle 1, 3 \rangle \) and \( \langle 3, 5 \rangle \) parallel?
2. Let \( \mathbf{a} = \langle -4, 1, 0 \rangle \) and \( \mathbf{b} = \langle 1, 0, 1 \rangle \).

(a) Find the vector projection \( \mathbf{p} = \text{proj}_\mathbf{b} \mathbf{a} \) of \( \mathbf{a} \) onto \( \mathbf{b} \).

Solution:

\[
\mathbf{a} \cdot \mathbf{b} = -4 + 0 + 0 = -4
\]

\[
\mathbf{b} \cdot \mathbf{b} = 1 + 0 + 1 = 2
\]

so

\[
\mathbf{p} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{b} \cdot \mathbf{b}} \mathbf{b} = \frac{-4}{2} \langle 1, 0, 1 \rangle = \langle -2, 0, -2 \rangle
\]

(b) With the vectors \( \mathbf{a} \) and \( \mathbf{p} \) from above, find the vector \( \mathbf{q} \) indicated in the following diagram:

\[
\mathbf{q} = \mathbf{a} - \mathbf{p} = \langle -4, 1, 0 \rangle - \langle -2, 0, -2 \rangle = \langle -2, 1, 2 \rangle
\]
3. Find the distance from the point \( P = (1, 1, 3) \) to the line \( x = t, y = 1 - 2t, z = 2 + 2t \).

Solution: A point on the line is \( Q = (0, 1, 2) \), and the vector from \( Q \) to \( P \) is \( \mathbf{a} = \langle 1, 0, 1 \rangle \).

A vector parallel to the line is \( \mathbf{v} = \langle 1, -2, 2 \rangle \).

We have

\[
\mathbf{a} \times \mathbf{v} = \langle 2, -1, -2 \rangle
\]

\[
\left( \mathbf{a} \times \mathbf{v} \right) \cdot \left( \mathbf{a} \times \mathbf{v} \right) = 4 + 1 + 4 = 9
\]

\[
\mathbf{v} \cdot \mathbf{v} = 1 + 4 + 4 = 9
\]

Thus the distance is

\[
\frac{\left\| \mathbf{a} \times \mathbf{v} \right\|}{\left\| \mathbf{v} \right\|} = \frac{\sqrt{9}}{\sqrt{9}} = 1
\]
4. (a) Find the line through the point \((1, 2, 3)\) and perpendicular to the plane \(3x - 2y + z = 7\).

Solution:
A normal vector to the plane is \( \langle 3, -2, 1 \rangle \), and this is also parallel to the desired line. Thus the line is \( x = 1 + 3t, y = 2 - 2t, z = 3 + t \).

(b) Find the plane through the point \((1, 2, 3)\) and parallel to both vectors \(2\mathbf{i} + \mathbf{j}\) and \(\mathbf{i} + \mathbf{j} + 2\mathbf{k}\).

Solution:
A vector perpendicular to both vectors is the cross product \( \langle 2, 1, 0 \rangle \times \langle 1, 1, 2 \rangle = \langle 2, -4, 1 \rangle \), and this is also perpendicular to the plane. Since \(2(1) - 4(2) + 1(3) = 2 - 8 + 3 = -3\), the plane is \(2x - 4y + z = -3\).
5. In parts (a) and (b) of this problem, give a rough sketch of the graph of the equation, and also identify it as one of the following types:

A: parabolic cylinder  D: elliptic paraboloid  G: ellipsoid
B: elliptic cylinder   E: hyperbolic paraboloid   H: hyperboloid of 1 sheet
C: hyperbolic cylinder F: cone       I: hyperboloid of 2 sheets

(a) $x^2 - y^2 + z^2 = 0$

(b) $x^2 - y^2 + z^2 = 5$
6. Consider the vector function \( \mathbf{r}(t) = (t, -t, t^2) \).

(a) Give a rough sketch of the curve defined by \( \mathbf{r}(t) \), indicating with an arrow the direction in which \( t \) increases.

(b) Find \( \int_0^2 \mathbf{r}(t) \, dt \).