Name:

- No books, notes or calculators of any kind are permitted on the test.
- Do not use your own scratch paper.
- Write each solution in the space provided, not on scratch paper.
- If you need more room, write on the back of the page. If you still need more room, ask for scratch paper.
- Be sure to give reasons for your answers (except where explicitly told otherwise)!
- Your solutions must be complete and organized, otherwise points may be deducted.
- There are 7 problems, and all problems have equal credit.
1. The parts of this problem are independent. 
   In this problem, only the answer will be graded; you do **not** need to show any work!

(a) Let \( f \) be a 2-dimensional scalar field. Which of the following two pictures could show \( \text{grad} f(P) \) and a level curve of \( f \) through \( P \)?

A: 

B: 

(b) Suppose that a moving particle has acceleration \( \mathbf{a} = (1, 2, 3) \) and velocity \( \mathbf{v} = (1, 0, -1) \) at some particular time. Which of the following describes what is happening to the speed at that moment?

A: increasing

B: decreasing

(c) True or False: if \( \frac{\partial f}{\partial x}(a, b) = 2 \), then \( f \) could have a local minimum at \( (a, b) \).

(d) True or False: the plane \( 2x - y + 3z = 4 \) is perpendicular to the vector \( (2, -1, 3) \).

(e) True or False: the straight line through \( (1, 2, 3) \) and \( (4, 5, 6) \) has parametric equations \( x = 1 + 4t, y = 2 + 5t, z = 3 + 6t \).

(f) What kind of paraboloid is the surface \( y = 2 - x^2 + z^2 \)?

A: elliptic

B: hyperbolic
2. The parts of this problem are independent.

(a) Stokes’ Theorem involves a vector field $\mathbf{F}$, an oriented surface $S$, and the boundary curve $C$, given a consistent orientation. Fill in the right-hand side of the following formula from this theorem:

$$\int_C \mathbf{F} \cdot d\mathbf{r} =$$

(b) The Fundamental Theorem for Line Integrals involves a scalar field $f$ and a curve $C$ with initial point $A$ and final point $B$. Fill in the right-hand side of the following formula from this theorem:

$$\int_C \nabla f \cdot d\mathbf{r} =$$

(c) Fill in the missing part of the following form of the Chain Rule: if $u$ is a function of $x, y$, and each of $x, y$ is a function of $s, t$, then

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial s} +$$
3. Let $\mathbf{F}$ be a vector field and $g$ a scalar field. In each part, indicate whether the result is a vector field, a scalar field, or is undefined:

(a) $\nabla \cdot (\nabla \times \mathbf{F})$
   A: vector field  B: scalar field  C: undefined

(b) $\nabla \times (\nabla g)$
   A: vector field  B: scalar field  C: undefined

(c) $\nabla \cdot (\nabla \times g)$
   A: vector field  B: scalar field  C: undefined

(d) $\nabla (\nabla \cdot \mathbf{F})$
   A: vector field  B: scalar field  C: undefined

(e) $\nabla \cdot (\nabla g)$
   A: vector field  B: scalar field  C: undefined

(f) $\nabla \times (\nabla \times \mathbf{F})$
   A: vector field  B: scalar field  C: undefined
4. Set up, but do not evaluate, an iterated integral in spherical coordinates for $\iiint_E xz \, dV$, where $E$ is the portion of the solid ball centered at the origin with radius 1 which lies in the 1st octant.
5. Let $\mathbf{F} = 3z\mathbf{k}$, and let $S$ be the closed surface made up of the part of the cylinder $x^2 + y^2 = 1$ between $z = 0$ and $z = 2$, and the parts of the planes $z = 0$ and $z = 2$ which lie inside the cylinder. **Use the Divergence Theorem** to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

Solution: The surface $S$ encloses the solid cylinder $E$: $x^2 + y^2 \leq 1; 0 \leq z \leq 2$; so $\iiint_S \text{div} \mathbf{F} \, dV = \iiint_E 3 \, dV = 3 \cdot \text{(volume of cylinder with radius 1 and height 2)} = 6$.
6. Use Green’s Theorem to find the value of the line integral

\[ \int_C xy \, dx + x^2 \, dy, \]

where \( C \) is the positively oriented closed curve in the \( xy \)-plane made up of:

- the straight line segment \( y = 0 \) with \( 0 \leq x \leq 1 \),
- the line segment \( x = 1 \) with \( 0 \leq y \leq 1 \),
- and the part of the parabola \( y = x^2 \) with \( 0 \leq x \leq 1 \).
7. Let $\mathbf{F} = (0, x, 0)$, and let $S$ be the portion of the plane $x + y + z = 1$ in the 1st octant, oriented upward. Set up, but do not evaluate, an iterated double integral for $\iint_S \mathbf{F} \cdot d\mathbf{S}$ by parameterizing in terms of $x$ and $y$.

**Do this directly** — do not attempt a clever application of the Divergence Theorem!