• Be sure to show all your work!
• Be sure to give reasons for your answers!
• Points will probably be taken off if you do not give a clear indication of
  the method and write your solution neatly, in order, and clearly indicate
  your final answer.
• Points may be taken off for a correct answer with a mistake in the sup-
  porting work.
• No books, notes or calculators of any kind are permitted on the test.
• Write each solution in the space provided, not on scratch paper.
• If you need more room, write on the back of the page.
• Time limit: 1.5 hours (90 minutes). Points may be deducted for exceeding
  this limit.
1. **In this problem only**, you do not need to give reasons, and only the answer will be graded:

(a) Find the linearization of the function \( f(x,y) = 3x + y^2 \) at the point \((1,1)\).

(b) A few contours for a function \( f \) are shown in the following diagram. Is \( \frac{\partial f}{\partial x} \) positive or negative at the point \( P \)?

(c) Write, but *do not evaluate*, an integral for the length of the curve
\[
\mathbf{r}(t) = (t,t^2,t^3), \quad 0 \leq t \leq 1
\]

(d) Which of the following is a contour map of \( f(x,y) = x + y^2 \)?

![Contour Maps]

A: ![Contour Map A]  B: ![Contour Map B]

(e) Is the function \( f(x,y,z) = \frac{x \ln y}{z^3} \) continuous at the point \((1,2,3)\)?

(f) The following diagram shows part of the path of a particle, as well as the unit tangent \( \mathbf{T} \), the unit normal \( \mathbf{N} \), and the acceleration \( \mathbf{a} \) at a point \( P \). Is the particle speeding up or slowing down at the point \( P \)?

![Diagram]
2. Consider a particle with position function \( \mathbf{r}(t) = (t, e^{2t}, \sin t) \). At \( t = 0 \), find the following:

(a) the velocity

(b) the acceleration

(c) the curvature
3. Let \( f(x, y) = \sin\left(\frac{\pi}{3} x + y\right) \), and let \( P \) be the point \( \left(1, 0, \frac{\sqrt{3}}{2}\right) \) and \( Q \) the point \( (1,0) \).

(a) Find the equation of the tangent plane to the surface \( z = f(x,y) \) at the point \( P \).

(b) Find the directional derivative of \( f \) at the point \( Q \) in the direction of the vector \( (1,1) \).
4. Consider the function \( f(x, y) = x^2 + y^2 + x^2y + 4 \).

(a) Show that the \((-\sqrt{2}, -1)\) is a critical point of \( f \)

(b) At the critical point \((-\sqrt{2}, -1)\), does \( f \) have a local maximum, local minimum, saddle point, or none of these?

(c) At the critical point \((0, 0)\), does \( f \) have a local maximum, local minimum, saddle point, or none of these? (You do not need to show that \((0, 0)\) is a critical point.)
5. Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint:

\[ f(x, y) = x^2 y; \quad x^2 + 2y^2 = 6 \]
6. Consider the following table of values of a continuous function \( f \) on the rectangle \( R = [0, 8] \times [1, 5] \):

<table>
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<th>( y )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>-2</td>
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<td>-2</td>
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<tr>
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<td>7.5</td>
<td>6</td>
<td>3.5</td>
<td>0</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

Use the Midpoint Rule with \( m = n = 2 \) to estimate \( \iint_R f(x, y) \, dA \).