1. (1 pt)
Consider the integral \( \int_0^{49} \int_0^{6\sqrt{x}} f(x,y) \, dy \, dx \). Sketch the region of integration and change the order of integration.
\[
\int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y) \, dx \, dy
\]
\( a = \quad b = \quad g_1(y) = \quad g_2(y) = \quad 
\)

2. (1 pt)
Consider the integral \( \int_0^1 \int_{8x}^{8x} f(x,y) \, dy \, dx \). Sketch the region of integration and change the order of integration.
\[
\int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y) \, dx \, dy
\]
\( a = \quad b = \quad g_1(y) = \quad g_2(y) = \quad 
\)

3. (1 pt) Find the volume of the solid enclosed by the paraboloids \( z = 1 \left( x^2 + y^2 \right) \) and \( z = 2 - 1 \left( x^2 + y^2 \right) \).

4. (1 pt)
Using polar coordinates, evaluate the integral \( \int \int_R \sin(x^2 + y^2) \, dA \) where \( R \) is the region \( 9 \leq x^2 + y^2 \leq 49 \).

5. (1 pt)
Suppose the solid \( W \) in the figure consists of the points below the \( xy \)-plane that are between concentric spheres centered at the origin of radii 2 and 10. Find the limits of integration for an iterated integral of the form
\[
\iiint_W f(\rho, \phi, \theta) \, d\rho \, d\phi \, d\theta
\]
\( A = \quad B = \quad C = \quad D = \quad E = \quad F = \quad 
\)

If necessary, enter \( \rho \) as rho, \( \phi \) as phi, and \( \theta \) as theta.
9. (1 pt) For each of the following vector fields \( \mathbf{F} \), decide whether it is conservative or not by computing the appropriate first order partial derivatives. Type in a potential function \( f \) (that is, \( \nabla f = \mathbf{F} \)) with \( f(0,0) = 0 \). If it is not conservative, type N.

A. \( \mathbf{F}(x, y) = (-4x - 5y) \mathbf{i} + (-5x + 10y) \mathbf{j} \)

\[ f(x, y) = \] ________

B. \( \mathbf{F}(x, y) = -2y \mathbf{i} - x \mathbf{j} \)

\[ f(x, y) = \] ________

C. \( \mathbf{F}(x, y) = (-2 \sin y) \mathbf{i} + (-10y - 2x \cos y) \mathbf{j} \)

\[ f(x, y) = \] ________

Note: Your answers should be either expressions of \( x \) and \( y \) (e.g. "3xy + 2y"), or the letter "N"

10. (1 pt) Consider the vector field \( \mathbf{F}(x, y, z) = xi + yj + zk \).

a) Find a function \( f \) such that \( \mathbf{F} = \nabla f \) and \( f(0,0,0) = 0 \).

\[ f(x, y, z) = \] ________

b) Use part a) to compute the work done by \( \mathbf{F} \) on a particle moving along the curve \( C \) given by \( \mathbf{r}(t) = (1 + 4 \sin t) \mathbf{i} + (1 + 4 \sin^2 t) \mathbf{j} + (1 + \sin^3 t) \mathbf{k} \), \( 0 \leq t \leq \frac{\pi}{2} \).

11. (1 pt) Consider the vector field \( \mathbf{F}(x, y, z) = (5z + 5y) \mathbf{i} + (z + 5x) \mathbf{j} + (y + 5x) \mathbf{k} \).

a) Find a function \( f \) such that \( \mathbf{F} = \nabla f \) and \( f(0,0,0) = 0 \).

\[ f(x, y, z) = \] ________

b) Suppose \( C \) is any curve from \((0,0,0)\) to \((1,1,1)\). Use part a) to compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

12. (1 pt)

Suppose \( \vec{F}(x, y) = (3x - 4y)\mathbf{i} + 3xy\mathbf{j} \)

and \( C \) is the counter-clockwise oriented sector of a circle centered at the origin with radius 3 and central angle \( \pi/3 \). Use Green’s theorem to calculate the circulation of \( \vec{F} \) around \( C \).

Circulation = ________________

13. (1 pt)

Use Green’s Theorem to evaluate the line integral of \( \mathbf{F} = \langle x^2, 6x \rangle \) around the boundary of the parallelogram in the following figure (note the orientation).

With \( x_0 = 2 \)

\[ \int_C x^2 \, dx + 6x \, dy = \] ________________

15. (1 pt) Compute the flux of the vector field \( \vec{F} = 8x^2y^2z^2 \) through the surface \( S \) which is the cone \( \sqrt{x^2+y^2} = z \), with \( 0 \leq z \leq R \), oriented downward.

(a) Parameterize the cone using cylindrical coordinates (write \( \theta \) as \( \text{theta} \)).

\[ x(r, \theta) = \] ________________

\[ y(r, \theta) = \] ________________

\[ z(r, \theta) = \] ________________

with \( _____ \leq r \leq _____ \)

and \( _____ \leq \theta \leq _____ \)

(b) With this parameterization, what is \( d\vec{A} \)?

\[ d\vec{A} = \] ________________

(c) Find the flux of \( \vec{F} \) through \( S \).

Flux = ________________

19. (1 pt) Determine whether each of the following vector fields appears to be path independent (conservative) or path dependent (not conservative).
20. (1 pt) Let $\mathbf{F} = (2xy, 4y^2)$ be a vector field in the plane, and $C$ the path $y = 4x^2$ joining $(0,0)$ to $(1,4)$ in the plane.

A. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$

B. Does the integral in part (A) depend on the path joining $(0,0)$ to $(1,4)$? ____ (y/n)

21. (1 pt) Consider the solid that lies above the square (in the $xy$-plane) $R = [0, 2] \times [0, 2]$, and below the elliptic paraboloid $z = 81 - x^2 - 2y^2$. Using iterated integrals, compute the exact value of the volume.

22. (1 pt) Evaluate the iterated integral $\int_0^2 \int_0^3 12x^2y^3 \, dxdy$

23. (1 pt) Evaluate the iterated integral $\int_1^2 \int_2^3 (x+y)^{-2} \, dydx$

24. (1 pt) Calculate the double integral $\int_R x \cos(2x+y) \, dA$ where $R$ is the region: $0 \leq x \leq \frac{\pi}{3}, 0 \leq y \leq \frac{\pi}{4}$

25. (1 pt) Evaluate the integral by reversing the order of integration.

\[
\int_0^1 \int_5^{5y} e^{-x} \, dxdy = \]

26. (1 pt) Consider the integral $\int_1^4 \int_0^{2\ln x} f(x,y) \, dydx$. Sketch the region of integration and change the order of integration.

\[
\int_a^b \int_{g_1(y)}^{g_2(y)} f(x,y) \, dxdy
\]

$g_1(y) = \phantom{0} \quad g_2(y) = \phantom{0}$