The equation of a line passing through \((x_0, y_0)\) with slope \(m\) is \(y = y_0 + m(x - x_0)\).

Often we replace \(y\) with a function name, say \(L(x)\), when we want to emphasize that the equation does define a function.

Example: \(\text{point} = (2, 3) \quad m = 6 \quad L(x) = 3 + 6(x - 2)\)

Example: \(\text{point} = (0, 0) \quad m = 3 \quad L(x) = 0 + 3(x - 0) \quad L(x) = 3x\)

The line tangent to the graph of \(y = f(x)\) at \(x = a\) is: \(\text{point} = (a, f(a)) \quad m = f'(a) \quad L(x) = f(a) + f'(a)(x - a)\).

Example: \(\sqrt{x} = x^2, \quad a = 2\)

\[
\begin{align*}
  f(x) &= x^2, \\
  f'(x) &= 2x, \\
  f(2) &= 4, \\
  f'(2) &= 4
\end{align*}
\]

\(L(x) = 4 + 4(x - 2)\).

Example: \(\frac{1}{\sqrt{x}}, \quad a = 4\)

\[
\begin{align*}
  f(x) &= \frac{1}{\sqrt{x}}, \\
  f'(x) &= -\frac{1}{2\sqrt{x}}, \\
  f(4) &= \frac{1}{2}, \\
  f'(4) &= -\frac{1}{2(2)} = -\frac{1}{4}, \\
  L(x) &= \frac{1}{2} + \left(-\frac{1}{4}\right)(x - 4)
\end{align*}
\]
\[ f(x) = \sqrt{x} \quad a = 25 \]
\[ f'(x) = \frac{1}{2\sqrt{x}} \]

\[ L(x) = 5 + (0.1)(x - 25) \]

\[ L(26) = 5 + (0.1)(26 - 25) = 5.100 \quad f(26) = 5.099 \]
\[ L(35) = 5 + (0.1)(35 - 25) = 6.000 \quad f(35) = 5.916 \]

\[ L(26) - f(26) = .001 \]
\[ L(35) - f(35) = .084 \]

\( L(x) \) is near \( f(x) \) if \( x \) is near 25.
2.3 Linear Approximation (Linearization)

Note: The "official" definition of the linear approximation of the function $f(x)$ at $x = a$ is the equation of the line tangent to the graph of $f(x)$ at the point $(a, f(a))$.

Thus the linear approximation (also called linearization) of $f(x)$ at $x = a$ is

$$L(x) = f(a) + f'(a)(x-a).$$

The significance of the linearization is the following:

If $x$ is "near" $a$, then $f(x) \approx L(x)$. 