Math 170 Review for Midterm 2

Topics: Chapter 2 and Chapter 3

(1) Find a formula for each function whose graph appears below.
(2) Consider the functions below.

\[ f(x) = x^2 - 4, \quad g(x) = \frac{2}{2 - x}, \quad h(x) = \frac{1}{\sqrt{x}}. \]

(a) Find \((f \circ g)(x)\).
(b) Find the domain of \((f \circ g)(x)\).
(c) Find \((g \circ f)(x)\).
(d) Find the domain of \((g \circ f)(x)\).
(e) Find \((h \circ f)(x)\).
(f) Find the domain of \((h \circ f)(x)\).

(3) (a) Let \(f(x) = x^2 + 2x\) and \(g(x) = x^2 + x\). Find \((f \circ g)(x)\) and find the domain of \((f \circ g)(x)\).
(b) Given the function \(h(x) = \sqrt{2x + 3}\). Find functions \(f(x)\) and \(g(x)\) such that \((f \circ g)(x) = h(x)\).

(4) Consider the function \(f(x) = \frac{x+1}{2x-3}\).
(a) Find the inverse function \(f^{-1}(x)\) of \(f(x)\).
(b) Find the domain of \(f(x)\).
(c) Find the domain of \(f^{-1}(x)\).
(d) Find the range of \(f(x)\).
(e) Find the range of \(f^{-1}(x)\).

(5) Consider the polynomial \(P(x) = 2x^3 - 7x^2 + 4x + 4\).
(a) Find all the possible rational zeros of \(P(x)\).
(b) Find all real zeros of \(P(x)\) and state their multiplicities.
(c) Find the behavior of \(P(x)\) as \(x \to \infty\) and \(x \to -\infty\).
(d) Sketch the graph of the function.

(6) Consider the rational function \(R(x) = \frac{2x^2 - 7x + 3}{1 - x^2}\).
(a) Find the vertical asymptotes of \(R(x)\) if there is any.
(b) Find the horizontal asymptote of \(R(x)\) if there is any.
(c) Find the x-intercepts of \(R(x)\) if there is any.
(d) Find the y-intercept of \(R(x)\) if there is any.
(e) Sketch the graph indicating vertical asymptotes, horizontal asymptote and x and y-intercepts of \(R(x)\).

(7) The curve below is the graph of a polynomial of degree 6 and passing through the point \((1, -\frac{5}{3})\). Find a formula for the polynomial.
(8) Find a possible formula for the rational function whose graph is given below.
(9) The number of bacteria in a culture at time $t$ is given by the formula

$$n = 10000\frac{3t^2 + 1}{t^2 + 1}.$$

(a) What is the initial population of bacteria?
(b) How long does it take for the initial bacteria population to double?
(c) Estimate the bacteria population after many years.

(10) (a) Find all the zeros, real and complex, of the polynomial function

$$x^4 - x^3 - 2x^2 + 6x - 4.$$

(b) Find a polynomial with integer coefficients with leading coefficient 1, degree 5, passing through the origin, and with zeros $i$ and $2 - i$. 