Tentative schedule

- **Lec 01** – Measure theory. \(\sigma\)-algebra, measurable space, measurable function, positive measure, construction of the abstract integral.
- **Lec 02** – Exercises 1.1, 1.5, 1.10 (Vitali sets, \(\sigma\)-additivity, inclusion-exclusion identity, matching problem) + 1.7.
- **Lec 03** – Exercises 1.11, 1.17, 1.21 (inclusion-exclusion identity, coupon collector’s problem) + 1.12.
- **Lec 04** – Monotone convergence theorem, Fatou’s lemma, dominated convergence theorem, Fubini’s theorem.

**HW 1** – Exercises 1.2, 1.8, 1.9, 1.14, 1.16.

- **Lec 05** – Radon-Nikodým theorem. Induced measure, change of variables formula, conditional expectation, independence.
- **Lec 06** – Exercises 2.4, 2.5, 2.10 (conditional probability, conditional expectation) + 2.12.
- **Lec 07** – Exercises 2.13, 2.14, 2.17 (the ballot problem, matching rounds problem) + 2.15.
- **Lec 08** – Limit theorems. Convergence of random variables, law of large numbers, central limit theorem. Exercise 3.2 (Weierstrass theorem).

**HW 2** – Exercises 2.3, 2.6, 2.8, 2.16, 3.4.

- **Lec 09** – Stochastic processes. General definition, filtration, martingales, Markov chains, basic examples: random walk, random walk with betting.
- **Lec 10** – Martingales. Stopping time, optional stopping theorem, application to the gambler’s ruin chain.
- **Lec 11** – Dubins’ inequality for the number of upcrossings, martingale convergence theorem, Kolmogorov’s zero-one law.
- **Lec 12** – Exercises 5.4, 5.5, 5.14. (gambler’s ruin chain, duration of a fair game) + 5.2.
- **Lec 13** – Branching processes. Model description, connection with martingales, extinction of the subcritical/critical process.
- **Lec 14** – Moment generating function of the offspring distribution, fixed point, probability of survival. Exercise 6.1 (shifted geometric distribution).

**HW 3** – Exercises 5.1, 5.6, 6.2, 6.3.

- **Lec 15** – Midterm exam.
- **Lec 16** – Review of the midterm exam.

- **Lec 17** – Discrete-time Markov chains. Matrix/graph representations, Chapman-Kolmogorov’s equations, communication classes, recurrence/transience.
- **Lec 18** – Stationary distribution, positive/null recurrence, period, existence and uniqueness of the stationary distribution, limiting probabilities, counter-examples.
- **Lec 19** – Exercises 7.6, 7.7, 7.9 (irreducibility, aperiodicity, existence and uniqueness of the stationary distribution, doubly stochastic matrix) + 7.4.
- **Lec 20** – Exercises 7.10, 7.11, 7.15 (probability of return, Ehrenfest chain) + 7.17.
- **Lec 21** – Symmetric random walks. Model description, recurrence in low dimensions, symmetry argument, transience in high dimensions.
- **Lec 22** – Exercises 8.1, 8.3, 8.6 (symmetric random walks on connected graphs, infinite collision property) + 8.2.

**HW 4** – Exercises 7.2, 7.5, 7.8, 8.4.

- **Lec 23** – Poisson point and Poisson processes. Spatial distribution, connection with the Poisson distribution, superposition and thinning properties.
- **Lec 24** – Temporal structure, connection with the exponential distribution, memoryless and basic properties of the exponential distribution, conditioning.
- **Lec 25** – Exercises 9.4, 9.5, 9.7 (queueing systems) + 9.9.

**HW 5** – Exercises 9.6, 9.8, 9.11, 9.12.

- **Lec 27** – Continuous-time Markov chains. Transition rates, intensity matrix, connection with Poisson processes and discrete-time Markov chains.
- **Lec 28** – Stationary distribution, Kolmogorov’s forward and backward equations, existence and uniqueness of the stationary distribution, limiting probabilities.
- **Lec 29** – Exercises 10.2, 10.3, 10.5 (fraction of time spent in a state, continuous-time random walks) + 10.6.
- **Lec 30** – Birth and death processes. Model description, recurrence/transience, existence and uniqueness of the stationary distribution, probability of survival.