Answers to HW set 2

• 1.5/5: You can differentiate $E$, integrate by parts (using the fact that since $u$ is stationary at the endpoints $u_t = 0$ at the end points) and obtain

$$dE/dt = \int_0^t (\rho_0 u_{tt} - \tau_0 u_{xx}) u_t \, dx = 0.$$ 

• 1.7/6: Let $v = w - u$. Note $v = 0$ on the boundary. In the integral

$$\int_{\Omega} |\nabla w|^2 \, dV$$

replace $\nabla w$ by $\nabla u + \nabla v$ and the above integral becomes

$$\int_{\Omega} |\nabla u|^2 \, dV + \int_{\Omega} |\nabla v|^2 \, dV.$$ 

• 1.8/3: $u = (12/\pi) \arctan(x) + 1$.

• 18/4: The general solution is $c_1 \ln(r) + c_2$. Apply the BC and $u = 10 \ln(r)/\ln(2)$.

• 1.9/2: Hyperbolic: $\xi = 2x + t$ and $\eta = t$;

$$u = \Phi(t) + \Psi(2x + t) \exp(t/4).$$ 

• 1.9/5: Elliptic. $\alpha = y + 3x$ and $\beta = \sqrt{3}x$. 