Answers to HW set 1

• 1.2/6:
  \[ u(x, t) = g(t - x/c) \exp(-\lambda x/c), \text{ if } x < ct, \]
  \[ u(x, t) = 0, \text{ if } x > ct \]

• 1.3/1
  \[ u_{xx} \approx 1/2, \quad u(6, T + 0.5) \approx 64.005 \]

• 1.3/2
  \[ E'(t) = -2k \int_0^\ell u_x^2 \, dx \]

• 1.3/3
  \[ v := g(t) + [h(t) - g(t)]x/\ell \]
  \[ w = u - v \text{ and } w_t - w_{xx} = -g'(t) + [g'(t) - h'(t)]x/\ell. \]

• 1.3/5
  \[ v := \frac{2k + 1}{2k} x - \frac{x^2}{2k} \]

• 1.3/7 Use a physical argument to show that the steady state cannot have a maximum above \( K \). Then look at the rate of change of the total population \( U(t) := \int_0^\ell u(x, t) \, dx \) to show that the solution must go to the constant function \( v(x) = K \).

• Extra problem 1.
  \[ u := -1 + (1 + \sin(-1 + \sqrt{1 + y + 2x})) \exp(x + 1 - \sqrt{1 + y + 2x}) \]