1. Solve the problem

\[ u_t = u_{xx}, \quad 0 < x < 1, \quad t > 0 \]
\[ u(x, 0) = f(x), \quad u_x(0, t) = 0, \quad u_x(1, t) = u(1, t). \]

2. Solve the problem

\[ u_{tt} - u_{xx} = 1, \quad 0 < x < L, \]
\[ u(0, t) = 3, \quad u_x(L, t) = 5, \]
\[ u(x, 0) = 0, \quad u_t(x, 0) = 0. \]

3. Solve the problem

\[ u_{tt} - c^2 u_{xx} = 0 \text{ on } 0 < x < \infty, \quad t > 0 \]
\[ u_x(0, t) = \phi(t), \text{ for } t > 0; \quad u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x) \text{ for } x > 0. \]

Here \( c \) is a positive constant.

4. Suppose \( \Delta u + f(r, \theta) = 0 \) on the disk of radius 1 (i.e. in the region \( r < 1 \)) with \( u(1, \theta) \leq 0 \), and suppose that \( M \) is a constant such that \( f(r, \theta) \leq 4M \). Use the maximum principle to show that \( u(r, \theta) \leq M(1 - r^2) \). Note that \( \Delta u = r^{-1}(ru_r)_r + r^{-2}u_{\theta\theta} \).

5. Use an energy integral to prove the uniqueness of bounded, finite energy solutions to the following Cauchy problem on \((-\infty, \infty)\) for \( t > 0 \):

\[ u_t = u_{xx} + f(x, t), \]
\[ u(x, 0) = \phi(x), \]
\[ u_x(x, t) \to 0 \text{ as } x \to \pm \infty. \]

6. Solve the Dirichlet problem:

\[ \Delta u = 0, \quad 1 < r < e, \quad 0 < \theta < \pi/2, \]
\[ u(1, \theta) = 0, \quad u(e, \theta) = 0, \quad u(r, 0) = 0, \quad u(r, \pi/2) = f(r). \]

Hint: \( r^{\pm i\omega} = e^{\pm i\omega \ln r} = \cos(\omega \ln r) \pm i \sin(\omega \ln r) \).