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3/2 STABILITY RESULTS FOR NONAUTONOMOUS STATE-DEPENDENT
DELAY DIFFERENTIAL EQUATIONS

YANG KUANG*
Department of Mathematics, Arizona State University
Tempe, AZ 85287-1804, USA

ABSTRACT

Sharp conditions for the boundedness of solutions, global and uniform stability of
the trivial solutions in a general class of state-dependent delay differential equations
are presented. These results are new even when restricted to various special cases
that have appeared in applications.

1. Introduction

We consider in this paper the following nonautonomous state-dependent delay
differential equation

\[ x'(t) = -g(t, x(t)) - e^{-\eta t} f(t, x(t - \tau(x))) \]

(1.1)

where \(\eta\) is a nonnegative constant and \(x_t(\theta) \equiv x(t + \theta), \) for \(\theta \in [-\tau, 0],\) where \(\tau\) is
described below. For \(g, \tau, f,\) we always assume the following: (H1) \(x g(t, x) \geq 0,\)
x\(f(t, x) > 0,\) if \(x \neq 0;\) \(g(t, x)\) and \(f(t, x)\) are continuous with respect to \(t,\) Lipschitz
and monotone increasing with respect to \(x.\) (H2) There exist nonnegative constants
\(k\) and \(\alpha\) such that for all \(t, x, x \neq 0,\)

\[ |g(t, x)/x| \leq k, \quad |f(t, x)/x| \leq \alpha. \]

(H3) \(\tau(\cdot) : C_\tau \to [\tau_m, \tau] \) where \(\tau > \tau_m \geq 0\) are constants. \(\tau(\cdot)\) satisfies that: for
any \(L > 0,\) there exist \(V = V(L) > 0\) such that if \(\varphi, \psi \in C_\tau, ||\varphi||, ||\psi|| \leq L,\) then

\[ |\tau(\varphi) - \tau(\psi)| \leq V \max_{-\tau \leq \theta \leq 0} |\varphi(\theta) - \psi(\theta)|. \]

Here \(C_\tau\) denotes the set of continuous functions defined on \([-\tau, 0]\) with the
standard norm \(||\varphi|| = \max_{-\tau \leq \theta \leq 0} |\varphi(\theta)|.\) In most applications, \(\tau(\varphi)\) has limits when
\(||\varphi|| \to 0\) or \(||\varphi|| \to +\infty,\) and \(\tau(0) = \tau.\) This is precisely the kind of state-dependent
delays that we shall consider here. The autonomous version of (1.1) has applications
in epidemiological modeling [9], immune response system study [1], and in
the investigation of respirational diseases [2,6]. For such autonomous equations,
Smith and Kuang [5] established conditions for the existence of nontrivial periodic
solutions. When \(g(t, x) = \nu x, \nu\) is a positive constant, \(\eta = 0\) and \(\tau(x) = \tau(x(t))\),
Eq. (1.1) is reduced to

\[ x'(t) = -\nu x - f(x(t - \tau(x(t))). \]

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This equation has been extensively studied by Mallet-Paret and Nussbaum [7], focusing on existence and properties of nontrivial periodic solutions. When \( g(t, x) \equiv 0, \eta = 0, \) and \( \tau(x, t) \equiv \tau, \) Eq. (1.1) reduces to
\[
x'(t) = -f(t, x(t - \tau)).
\] (1.2)

This equation has been extensively studied (see Kuang [3], §4.5). It is well known that if \( \alpha \tau < 3/2, \) then the trivial solution of (1.2) is uniformly stable, while if \( \alpha \tau > 3/2, \) the trivial solution of (1.2) can be unstable due to an example of Yoneyama [10]. Such kinds of results are often called 3/2 stability results and generally viewed as sharp. Recently, by employing a rather novel approach, Kuang [4] was able to show that if in addition to (H1)–(H2), \( \alpha \tau e^{\kappa \tau} < 3/2, \) then the trivial solution of the following nonautonomous distributed delay differential equation
\[
x'(t) = -g(t, x(t)) - f\left(t, \int_{-\tau}^{0} x(t + s) d\mu(s)\right),
\] (1.3)
is uniformly stable where \( \mu(s) \) is nondecreasing on \([-\tau, 0]\) and \( \mu(0) - \mu(-\tau) = 1. \)

If, in addition to the above conditions, there is a continuous function \( l(t) \) such that \( \int_{0}^{\infty} l(t) dt = +\infty, \) and
\[
|f(t, x)/x| \geq l(t), \quad x \neq 0,
\]

then the trivial solution of (1.3) is globally asymptotically stable. It is the purpose of this paper to obtain similar results for state-dependent delay differential equations that are at least as general as (1.1). Our approach here will be similar to that of Kuang [4]. However, there are several subtle yet important places that require novel treatments.

2. Preliminaries and Boundedness

In the rest of this paper we always assume that (1.1) satisfies \( x_{t_0} = \varphi, \varphi \in C_{\tau}, \) and \( \varphi \) is Lipschitzian. Then it is well known (see Lemma 1.1 of Smith and Kuang [5]) that under assumptions (H1) and (H3), the solution of (1.1) is unique. The following lemma shows that under assumptions (H1)–(H3), the solution is defined for all \( t \geq t_0 \) and is bounded by some exponential function. In the following, we denote \( x(t_0, \varphi) \) the solution of (1.1) with \( x_{t_0} = \varphi, \) and \( x(t) \equiv x_{t}(t_0, \varphi(0)), t \geq t_0. \)

**Lemma 2.1.** For all \( t \geq t_0, \) the solution \( x(t_0, \varphi) \) of (1.1) is defined and satisfies
\[
|x(t)| \leq ||\varphi|| \exp\{(k + \alpha)(t - t_0)\}. \] (2.1)

**Proof.** From Lemma 1.1 of Smith and Kuang [5], we know that if \( x_{t} \) is a noncontinuable solution of (1.1) defined for \( t \in [t_0, \omega), \omega < +\infty, \) then \( \limsup_{t \rightarrow \omega} |x(t)| = +\infty. \)

So, if we can show that (2.1) is true for \( t \in [t_0, \omega), \) then \( x_{t} \) can be extended to \( \omega, \)
which implies that $x_t$ must exist for all $t \geq t_0$ and (2.1) is true for all $t \geq t_0$, proving
the lemma. Denote

$$z(t) = \max\{|x(s)| : s \in [t_0 - \tau, t]\}.$$ 

Observe that if $|x(t)| \neq z(t)$, then $z'(t) = 0$, and if $|x(t)| = z(t)$, then $z'_+(t) = |x'(t)|$,
where $z'_+(t)$ denotes the right derivative of $z(t)$. We have

$$z'_+(t) \leq |x'(t)| \leq k|x(t)| + \alpha|x(t - \tau(x_t))| \leq (k + \alpha)z(t),$$

which yields $z(t) \leq ||\varphi||e^{(k+\alpha)(t-t_0)}$, $t \geq t_0$. Clearly, $|x(t)| \leq z(t)$, which leads to
(2.1). This proves the lemma. □

With one additional assumption, we can show that solutions of (1.1) are bounded. In the rest of this paper, we denote

$$\tau^\infty = \limsup_{||\phi|| \to \infty} \tau(\phi), \quad \phi \in C_\tau.$$ 

**Theorem 2.1.** Assume that (1.1) satisfies (H1)–(H3) and

$$\alpha e^{k\tau - \eta \tau_m} \tau^\infty < 3/2$$

Then solutions of (1.1) are bounded.

**Proof.** For the sake of contradiction, we assume that $x_t(t_0, \varphi)$ is not bounded for some $\varphi \in C_\tau$. Then we have two cases to consider (1) there is a $T \geq t_0 + \tau$ such that for $t \geq T$, $x(t) \neq 0$. (2) $x(t)$ has arbitrary large zeros. In case (1), it is easy to see from the monotone increasing property of $f$, $g$, that for $t \geq T + \tau$,

$$|x(t)| \leq |x(T + \tau)|.$$ 

Hence, $x_t$ must be bounded. We consider now case (2). Observe that (1.1) can be rewritten as

$$y'(t) = -\exp\left(\int_{t_0}^{t} \frac{g(s, x(s))}{x(s)} ds\right) e^{-\eta \tau(x_t)} f(t, x(t - \tau(x_t)))$$

where

$$y(t) = x(t) \exp\left(\int_{t_0}^{t} \frac{g(s, x(s))}{x(s)} ds\right).$$

(2.4)

Clearly, $y(t)$ must also have arbitrary large zeros. Let $\epsilon > 0$, $M^* > 0$ such that

$$\sup\{\tau(\phi) : ||\phi|| \geq M^*/2\} < \tau^\infty + \epsilon.$$ 

(2.5)

$$\alpha(\tau^\infty + \epsilon) \exp(k\tau - \eta \tau_m) < 3/2.$$ 

(2.6)

Assume that $x(t)$ is unbounded. Then, without loss of generality, we may assume that there is a $\bar{t} \geq 2\tau$ such that $x(\bar{t}) \geq M^*$, $x'(') \geq 0$, $|x(t)| < M^*$ for $t \in [t_0, \bar{t})$. Then it is easy to see from (2.4) that we must have

$$y'(\bar{t}) \geq 0, \quad |y(t)| < y(\bar{t}) \equiv M, \quad \text{for} \quad t \in [t_0, \bar{t}).$$

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\( y'(\check{t}) \geq 0 \) and (2.3) imply that \( f(\check{t}, x(\check{t} - \tau(x_{\sigma}))) \leq 0 \), and hence \( x(\check{t} - \tau(x_{\sigma})) \leq 0 \). Denote

\[
\check{t} = \sup \{ t : t < \check{t}, x(t) = 0 \}
\]

\[
\check{t} = \sup \{ t : t \in [t_*, \check{t}], y(t) = \frac{1}{2} M \}.
\]

For \( t \in [t_*, \check{t}] \), we have

\[
|y(t)| = |y(t) - y(t_*)| = \left| \int_{t_*}^{t} y(\sigma) d\sigma \right|
\]

\[
\leq \left| \int_{t_*}^{t} \exp \left( \int_{t_*}^{\sigma} g(s, x(s)) ds \right) e^{-\eta \tau(x_{\sigma})} |f(\sigma, x(\sigma - \tau(x_{\sigma})))| d\sigma \right|
\]

\[
\leq \alpha e^{-\eta \tau} \left| \int_{t_*}^{t} \exp \left( \int_{t_*}^{\sigma} g(s, x(s)) ds \right) |x(\sigma - \tau(x_{\sigma}))| d\sigma \right|
\]

\[
\times \exp \left( \int_{t_*}^{\sigma - \tau(x_{\sigma})} \frac{g(s, x(s))}{x(s)} ds \right) d\sigma
\]

\[
\leq \alpha Me^{-\eta \tau} \left| \int_{t_*}^{t} \exp (k\tau(x_{\sigma})) d\sigma \right| \leq \alpha Me^{k\tau - \eta \tau} |t - t_*|. \quad (2.7)
\]

In particular, we have

\[
\alpha e^{k\tau - \eta \tau} (\check{t} - t_*) \geq \frac{1}{2}. \quad (2.8)
\]

Note that for \( \sigma \in [\check{t}, \check{t}], x(\sigma - \tau(x_{\sigma})) \leq 0 \) implies that \( \sigma - \tau(x_{\sigma}) \leq t_* \). Similar to (2.7) and using (2.7), we shall obtain the following

\[
\frac{1}{2} M = y(\check{t}) - y(\check{t}) = \int_{\check{t}}^{t} y'(t) dt
\]

\[
\leq \alpha e^{-\eta \tau} \left| \int_{\check{t}}^{t} \exp \left( \int_{\check{t}}^{\sigma} g(s, x(s)) ds \right) \left[ -x(\sigma - \tau(x_{\sigma})) \right]_+ \right|
\]

\[
\times \exp \left( \int_{t_*}^{\sigma - \tau(x_{\sigma})} \frac{g(s, x(s))}{x(s)} ds \right) d\sigma
\]

\[
\leq \alpha^2 M e^{2k\tau - 2\eta \tau} \left| \int_{\check{t}}^{t} \left[ t_* + \tau(x_{\sigma}) - \sigma \right]_+ d\sigma \right|, \quad (2.9)
\]

where \([x]_+ = x \) if \( x \geq 0 \) and \([x]_+ = 0 \) if \( x < 0 \). Since \( \check{t} \in [t_*, \check{t}] \), we have \( \check{t} - \check{t} < \tau \).

For \( t \in [\check{t}, \check{t}] \) we have \( y(t) \geq y(\check{t})/2 \). Hence, for \( t \in [\check{t}, \check{t}] \),

\[
x(t) = y(t) \exp \left( - \int_{t_0}^{t} \frac{g(s, x(s))}{x(s)} ds \right)
\]

\[
\geq \frac{1}{2} y(\check{t}) \exp \left( - \int_{t_0}^{\check{t}} \frac{g(s, x(s))}{x(s)} ds \right) \exp \left( \int_{\check{t}}^{t} \frac{g(s, x(s))}{x(s)} ds \right)
\]

\[
\geq M^*/2.
\]
This implies that for $t \in [\hat{t}, \bar{t}]$,

$$\tau(x_t) < \tau^\infty + \epsilon.$$  

Observe that for $\sigma \in [\hat{t}, \bar{t}]$, $\sigma \leq \bar{t} < t_* + \tau^\infty + \epsilon$. We have for $\sigma \in [\hat{t}, \bar{t}]$,

$$[t_* + \tau(x_\sigma) - \sigma]_+ \leq [t_* + \tau^\infty + \epsilon - \sigma]_+ = t_* + \tau^\infty + \epsilon - \sigma. \quad (2.10)$$

Hence, (2.9) yields

$$\frac{1}{2} M \leq \frac{1}{2} \alpha^2 M e^{2kT - 2\eta \tau_m} [(t_* + \tau^\infty + \epsilon - \hat{t})^2 - (t_* + \tau^\infty + \epsilon - \bar{t})^2],$$

which leads to

$$\alpha e^{kT - \eta \tau_m} (t_* + \tau^\infty + \epsilon - \hat{t}) \geq 1. \quad (2.11)$$

Combining (2.8) with (2.11), we obtain

$$\alpha e^{kT - \eta \tau_m} (\tau^\infty + \epsilon) \geq 3/2,$$

a contradiction to (2.6). This shows that solution $x(t)$ must be bounded in case (2), proving the theorem. \qed

The following corollary is obvious from Theorem 2.1.

Corollary 2.1. Assume $g(t, \cdot) \equiv 0$, $\eta = 0$ in (1.1), and $\alpha \tau^\infty < 3/2$, then solutions of (1.1) are bounded. The above corollary clearly improves and generalizes the following proposition (2.2) of Kuang and Smith (1992) in several important aspects.

Proposition 2.2 (Kuang and Smith [5]). Assume $f(x)$ and $\tau(x)$ are locally lipschitzian and (i) $\tau_\pm = \lim_{x \to \pm \infty} \tau(x)$ exist, $\lim_{|x| \to \infty} x\tau'(x) = 0$; (ii) $\alpha \tau^\infty < 3/2$, where $\tau^\infty = \max\{\tau_\pm\}$, $\alpha = \sup\{|f(x)/x| : x \neq 0\}$. Then solutions of $x' = -f(x(t - \tau(x(t))))$ with lipschitzian initial functions are bounded.

3. Stability Results

From (H3) we see that $\tau(\varphi)$ has limit when $||\varphi|| \to 0$. In the rest of this paper, we denote

$$\tau_0 = \lim_{||\varphi|| \to 0} \tau(\varphi).$$

Recall that we say the trivial solution of (1.1) is uniformly stable if for any $\epsilon > 0$ there is a $\delta = \delta(\epsilon)$, such that $\varphi \in C_B$ and $||\varphi|| < \delta$ implies that $||x_t(t_0, \varphi)|| < \epsilon$, for $t \geq t_0$. The proof of the following theorem is essentially the same as that of Theorem 2.1.

Theorem 3.1. Assume that (H1)-(H3) hold for equation (1.1), $\tau_0 > 0$, and

$$\alpha \tau_0 e^{(k-\eta)\tau_0} < 3/2, \quad (3.1)$$

then the trivial solution of (1.1) is uniformly stable.
Proof. Given $\epsilon > 0$ we can find $\epsilon_0, \epsilon_1$, such that $0 < \epsilon_0 < \epsilon_1 < \min\{\tau, \epsilon\}$, and for 
$||\phi|| \leq \epsilon_0$, $\tau(\phi) \in (\tau_0 - \epsilon_1, \tau_0 + \epsilon_1)$, and

$$\alpha(\tau_0 + 3\epsilon_1) \exp\{k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)\} < 3/2. \tag{3.2}$$

We define

$$\delta = \epsilon_0 \exp\{-(k + \alpha)\tau\}. \tag{3.3}$$

We shall show that $\phi \in C_B$ and $||\phi|| < \delta$ implies that $||x_t(t_0, \phi)|| < \epsilon_1(\epsilon_0)$ for $t \geq t_0$. From Lemma 2.1, we have for $t \leq t_0 + \tau,$

$$||x_t(t_0, \phi)|| < \delta e^{(k+\alpha)\tau} = \epsilon_0.$$

Assume the theorem is false; then there is a $\hat{t} > t_0 + \tau$ such that, without loss of generality,

$$x'(\hat{t}) \geq 0, \quad x(\hat{t}) = \epsilon_0, \quad |x(t)| < \epsilon_0, \quad \text{for } t \in [t_0, \hat{t}). \tag{3.4}$$

We define $y(t)$ as in (2.4) and $M, t_*$, $\hat{t}$ similarly. Similar to (2.7) we shall obtain that for $t \in [t_* - \tau, \hat{t}]$,

$$|y(t)| \leq \alpha Me^{k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)}|t - t_*|. \tag{3.5}$$

Similarly, we shall obtain, like (2.9),

$$\frac{1}{2}M = y(\hat{t}) - y(\hat{t}) \leq \alpha^2 Me^{2k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)} \int_{\hat{t}}^{\hat{t}} |\sigma - \tau(x_\sigma) - t_*| d\sigma. \tag{3.6}$$

Since $|x(\sigma)| \leq \epsilon_0$ for $\sigma \in [\hat{t} - \tau, \hat{t}]$, we have for $\sigma \in [\hat{t}, \hat{t}]$ $|\tau_0 + \epsilon_1 - \tau(x_\sigma)| \leq 2\epsilon_1$, $\sigma \leq \hat{t} \leq t_* + \tau_0 + \epsilon_1$. Hence,

$$\int_{\hat{t}}^{\hat{t}} |\sigma - \tau(x_\sigma) - t_*| d\sigma \leq \int_{\hat{t}}^{\hat{t}} |\sigma - \tau_0 - \epsilon_1 - t_*| + |\tau_0 + \epsilon_1 - \tau(x_\sigma)| d\sigma$$

$$\leq \int_{\hat{t}}^{\hat{t}} (t_* + \tau_0 + 3\epsilon_1 - \sigma) d\sigma < \frac{1}{2}(t_* + \tau_0 + 3\epsilon_1 - \hat{t})^2. \tag{3.7}$$

Hence, by (3.6) and (3.7) we obtain

$$\alpha e^{k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)}(t_* + \tau_0 + 3\epsilon_1 - \hat{t}) > 1. \tag{3.8}$$

Letting $t = \hat{t}$ in (3.5), we have

$$\alpha e^{k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)}(\hat{t} - t_*) \geq \frac{1}{2}. \tag{3.9}$$

Combining (3.8) and (3.9), we obtain

$$\alpha e^{k(\tau_0 + \epsilon_1) - \eta(\tau_0 - \epsilon_1)}(\tau_0 + 3\epsilon_1) > 3/2,$$

a contradiction to (3.2). This proves the theorem. \qed

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Theorem 3.2. Assume that (H1)-(H3) hold for equation (1.1) and there is a non-negative continuous function \( l(t) \), \( \int_0^\infty l(t) \, dt = +\infty \), such that for \( x \neq 0 \), \(|f(t,x)/x| \geq l(t)\), and
\[
\alpha e^{k_{\tau - \eta_m}} < \frac{3}{2}. \tag{3.10}
\]
Then the trivial solution of (1.1) is globally asymptotically stable. That is,
\[
\lim_{t \to \infty} x(t) = 0.
\]

Proof. Since \( \tau^\infty \leq \tau \), we see that \( x(t) \) is bounded. Denote
\[
u = \limsup_{t \to \infty} |x(t)|,
\]
then \( \nu \) is finite. The conclusion of the theorem is equivalent to \( \nu = 0 \) for all \( x(t) \). Assume for the sake of contradiction that \( \nu > 0 \). Then we can find \( \epsilon \in (0, \nu/4) \) such that
\[
\alpha \tau e^{k_{\tau - \eta_m}} < \frac{1}{2} + \sqrt{\frac{\nu - 3\epsilon}{\nu + \epsilon}}. \tag{3.11}
\]
If \( x(t) \) is eventually monotone, say, without loss of generality, \( x(t) \to \nu \), then for large \( t \), \( x'(t) < -e^{-\eta_m} l(t) x(t - \tau(x)) \leq -e^{-\eta_m} l(t) \nu \), which implies that \( x(t) \to -\infty \), a contradiction to the fact that \( x(t) \to \nu \). So we assume below that \( x(t) \) has arbitrarily large zeros. By the definition of \( \nu \) and the choice of \( \epsilon \), we see that there is a \( T > t_0 + \tau \) such that \( t \geq T \) implies that \( |x(t)| < \nu + \epsilon \). Without loss of generality, we assume that there is a \( t, \tilde{t} \geq T + 2\tau \),
\[
x(\tilde{t}) \geq \nu - \epsilon, \quad x'(\tilde{t}) \geq 0. \tag{3.12}
\]
Define \( y(t) \) as in (2.4); we have \( y'(\tilde{t}) \geq 0 \). Denote \( M = y(\tilde{t}) \) and
\[
M = \max\{y(t) : t \in [\tilde{t} - 2\tau, \tilde{t}]\}. \tag{3.13}
\]
It is easy to see that
\[
\overline{M} \geq \frac{(\nu - \epsilon)/(\nu + \epsilon)}{\nu} M. \tag{3.14}
\]
Defining \( t_*, \hat{t} \) as in the proof of Theorem 2.1, we shall obtain that for \( t \in [t_* - \tau, \hat{t}] \),
\[
|y(t)| \leq \alpha M e^{k_{\tau - \eta_m}} |t - t_*|. \tag{3.15}
\]
In particular, we have
\[
\alpha e^{k_{\tau - \eta_m}} (\hat{t} - t_*) \geq \frac{1}{2}. \tag{3.16}
\]

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Similarly, we shall obtain that 
\[
\frac{1}{2} M \frac{u - 3\varepsilon}{u + 3\varepsilon} \leq y(\hat{t}) - y(\hat{t}) \\
\leq \alpha^2 Me^{2k\tau - 2\eta \tau_m} \int_{t}^{\hat{t}} [t_* + \tau(x_\sigma) - \sigma]_+ d\sigma \\
\leq \frac{1}{2} \alpha^2 Me^{2k\tau - 2\eta \tau_m} (t_* + \tau - \hat{t})^2,
\]
which implies that 
\[
\alpha e^{k\tau - \eta \tau_m} (t_* + \tau - \hat{t}) \geq \sqrt{\frac{u - 3\varepsilon}{u + \varepsilon}}. \tag{3.17}
\]
Combining (3.16) with (3.17), we obtain 
\[
\alpha \tau e^{k\tau - \eta \tau_m} \geq \frac{1}{2} + \sqrt{\frac{u - 3\varepsilon}{u + \varepsilon}},
\]
which contradicts (3.11). This shows that \( u \) must be zero, proving the theorem.
\[\square\]

When \( \tau(x_i) \equiv \tau \), (3.10) is reduced to \( \alpha \tau e^{(k-\eta)\tau} < \frac{3}{2} \). Even in this special case, Theorem 3.2 is new. If, in addition, \( k = \eta = 0 \), then (3.10) is simplified as \( \alpha \tau < \frac{3}{2} \) and Theorem 3.2 becomes the well-known \( 3/2 \) stability result that is generally viewed as sharp, since the number \( 3/2 \) cannot be enlarged.

4. References

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