Lectures 12, Th., Sept. 28

Reading homework: DCDS-B article: Is there a sigmoid growth of Gause’s paramecium caudatum in constant environment, by Zufei Ma, Dianmo Li and Baoyu Xie. It can be downloaded at http://aimsciences.org/journals/pdfs.jsp?paperID=541&mode=full

1. Two-point cycles. We briefly covered sections 2.4 and part of 2.5 of the textbook.

The composition method of studying 2-point cycle can be useful in some interesting applications. For example, we can consider a population that consumes its own juveniles in order to self-regulate its size. Assume that the encounter of juvenile and adult individuals follow a Poisson distribution, then the probability that a juvenile will escape from being cannibalized is $e^{-cx_2}$ when the adult prey population is of size $x_2$ for some $c > 0$, where $c$ may be interpreted as the cannibalism rate. The birth rate of the adult population is assumed to be a constant $b > 0$. The survival probability of the juvenile population is density dependent and depends only on its own population size. For simplicity, a Beverton-Holt type (Holling type II) function can be used to model this nonlinearity. Then a plausible model may take the form of

$$
\begin{align*}
    x_1(t+1) &= bx_2(t) e^{-cx_2(t)}, \\
    x_2(t+1) &= \frac{ax_1(t)}{1 + mx_1(t)}.
\end{align*}
$$

This model can be reduced to a scalar nonlinear difference equation and the results of lecture note 8 can be applied.

2. Gause’s experiments revisited. We briefly covered the paper “Is there a sigmoid growth of Gause’s paramecium caudatum in constant environment”, by Zufei Ma, Dianmo Li and Baoyu Xie. It can be downloaded at http://aimsciences.org/journals/pdfs.jsp?paperID=541&mode=full