1. (5 pts) State the definition of the double integral \( \iint_R f(x,y)\,dA \) where \( R = [a,b] \times [c,d] \).

\[
\iint_R f(x,y)\,dA = \lim_{n,m \to \infty} \sum_{i=1}^n \sum_{j=1}^m f(x_i^*, y_j^*) \cdot \Delta A
\]

where \( \Delta A = \frac{b-a}{n} \cdot \frac{d-c}{m} \)

2. (10 pts) Find the average value of \( f(x,y) = y\cos(x) + 4 \) on \( R = [0, \pi/6] \times [4,5] \).

\[
\text{fave} = \frac{\iint_R f(x,y)\,dA}{\text{area}(R)} = \frac{1}{\pi/6} \int_4^5 y\cos x + 4 \,dy \,dx = \frac{1}{\pi/6} \int_0^{\pi/6} \left[ \frac{1}{2} y^2 \cos x + 4y \right]_4^5 \,dx
\]

\[
= \frac{1}{\pi/6} \left[ \frac{25}{2} \cos x + 20 - \left( \frac{16}{2} \cos x + 16 \right) \right]_0^{\pi/6} = \frac{6}{\pi} \left( \frac{9}{4} + \frac{4\pi}{6} \right) = \frac{2\pi}{3\pi} + 4
\]

3. (15 pts) FIND \( \iint \frac{1}{\sqrt{1+x^2+y^2}} \,dy\,dx \) using polar coordinates.

\[
x = r\cos \theta \quad \rightarrow \quad r^2 = x^2 + y^2
\]

\[
y = r\sin \theta
\]

\[
\begin{align*}
&1 \quad 0 \leq r \leq 1 \\
&0 \leq \theta \leq \pi/2
\end{align*}
\]

\[
\iint \frac{1}{\sqrt{1+r^2}} \,dy\,dx = \iint \frac{1}{\sqrt{1+r^2}} \cdot r \,dr\,d\theta
\]

\[
= \frac{1}{2} \int_{\pi/2}^1 \frac{1}{\sqrt{u}} \,du \,\,d\theta = \frac{1}{2} \int_{\pi/2}^1 2\sqrt{u} \,du \,\,d\theta = \int_0^{\pi/2} \left( \sqrt{u} - 1 \right) \,d\theta = \theta \left( \sqrt{u} - 1 \right)
\]

\[
= \frac{\pi}{2} (\sqrt{u} - 1)
\]
• NO TI-89 OR HIGHER ability CALCULATORS ARE ALLOWED!!! Your calculator should not have any supporting program on it!

4. (12 pts) SET UP (do not calculate!!!) a triple integral to find the volume of the finite solid bounded by $2x + y + z = 4$ and the coordinate planes.

\[ V = \iiint_{0}^{2} \iiint_{0}^{4-2x} \iiint_{0}^{4-2x-y} 1 \, dz \, dy \, dx \]

5. (15 pts) Evaluate the integral: $\int_{0}^{4} \int_{\sqrt{x}}^{x^2} e^x \, dy \, dx$. (Be careful that you do not invent new, non-existing rules of integration!!!)

\[ \text{Rewrite as region type I} \]

\[ \int_{0}^{1} \int_{0}^{x^2} e^x \, dy \, dx = \int_{0}^{1} e^x \, dy \left[ 0 \right]_{0}^{x^2} \]

\[ = \int_{0}^{1} e^x \, dx = \left. \frac{1}{3} e^u \right|_{0}^{1} = \frac{1}{3} (e - 1) \]
• NO TI-89 OR HIGHER ability CALCULATORS ARE ALLOWED!!! Your calculator should not have any supporting program on it!

6. (10 pts) The figure shown is bounded by \( x = 36 - 4y^2, \ y = z \) and the \( xy, yz \) planes. Put appropriate limits on the triple integral shown in the order given.

\[
0 \leq 36 - 4y^2 \leq 3 \\
0 \leq y \leq \sqrt{\frac{36}{4}} = 3 \\
0 \leq z \leq y
\]

7. (15 pts) SET UP an integral to evaluate \( \iint f(x, y) \, dx \) over the region bounded by \( y = 2x, \ y = x+1 \) and the \( y \) axis.

(a) using Type I region
\[
\int_0^{2x} \int_1^{x+1} f(x, y) \, dy \, dx
\]

(b) using Type II region
\[
\int_0^{1/2} \int_0^{y} f(x, y) \, dx \, dy + \int_{1/2}^{y/2} \int_0^{y-1} f(x, y) \, dx \, dy
\]

(c) calculate one of the above integrals.
\[
\int_0^{2x} \int_0^{x+1} y \, dy \, dx = \int_0^{2x} \frac{1}{2} y^2 \bigg|_0^{x+1} \, dx = \frac{1}{2} \int \left( (x+1)^2 - 4x^2 \right) \, dx
\]
\[
= \frac{1}{2} \int \left( x^2 + 2x + 1 - 4x^2 \right) \, dx = \frac{1}{2} \left( \frac{1}{3} x^3 + x + \frac{1}{2} \right) \bigg|_0^{1/2} = \frac{1}{8}
\]
8. (10 pts) Rewrite but do not calculate the integral \( \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} (x^2 + y^2)^{3/2} \, dz \, dy \, dx \) in cylindrical coordinates.

\[
\begin{align*}
\int_{0}^{2\pi} & \int_{0}^{2} \int_{0}^{r^2} (r^2)^{3/2} \, r \, dz \, dr \, d\theta \\
& \downarrow \text{region in xy plane} \\
\end{align*}
\]

9. (10 pts) Set up but do not calculate the integral \( \iiint_{B} e^{\sqrt{x^2+y^2+z^2}} \, dV \) in spherical coordinates where \( B \) is the region bounded by the spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 9 \).

\[
\begin{align*}
\iiint_{B} & e^{\sqrt{x^2+y^2+z^2}} \, dV \\
& \downarrow \\
& \begin{align*}
& x = r \cos \theta \sin \phi \\
y = r \sin \theta \sin \phi \\
z = r \cos \phi \\
& \Rightarrow x^2 + y^2 + z^2 = r^2 \\
& 2 \leq r \leq 3 \\
& 0 \leq \theta \leq 2\pi \\
& 0 \leq \phi \leq \pi \\
& \text{Jacobian} = r^2 \sin \phi \\
\end{align*}
\]

10. (5 pts) Describe the solid over which the triple integral is taken: \( \int_{0}^{2\pi} \int_{0}^{\sqrt{1-r^2}} \int_{0}^{\sqrt{1-x^2-y^2}} rdz \, dr \, d\theta \). (thinking in rectangular coordinates will help).

because of \( \sqrt{1-r^2} \) only positive
(top) half of the sphere

\[
\begin{align*}
& 0 \leq z \leq \sqrt{1-r^2} \\
& z^2 = 1 - x^2 - y^2 \\
& x^2 + y^2 + z^2 = 1 \text{ hemi sphere}
\end{align*}
\]