Practice Set for Section 6.4

Title: Extrema of Surfaces with Boundaries

For practice, work these by hand as in the traditional manner.

1. Let \( f(x, y) = x^2 + y^2 + xy - 5x - 4y + 1 \) over the square constraint region defined by \( S : -4 \leq x \leq 4, -4 \leq y \leq 4 \). Find the maximum value of the function over the constraints (the region of feasible solutions).

2. Find the minimum value of the function in problem 1 subject to the same constraints.

3. Let the region of feasible solutions be given by the following constraints: \( x \geq 0, y \geq 0, x \leq 4, x + y \leq 6 \). Find the minimum and maximum values that the surface \( f(x, y) = x^2 + y^2 + xy - 4x - 5y + 2 \) achieves over this region.

A t-shirt vendor sells two types of shirts, and his profit function is given by the function \( g(x, y) = -x^2 - y^2 - xy + 50x + 55y + 125 \), where \( x \) and \( y \) represent the two different types of shirts, and \( g \) is the profit in dollars per week. Shirt “x” requires 1 hour of machine time to make, and shirt “y” requires 3 hours on the machine to make, and he has at most 60 hours of machine time per week available to make his shirts.

4. Ignoring any constraints, how many of shirt “x” and of shirt “y” should he sell to maximize his profit for any given week?

5. Now consider the constraints. Create equations for the region of feasibility (recall that he can’t produce and sell negative quantities of shirts). Sketch this region. Where does the critical point in problem 4 lie? Inside this region or outside?

6. Find the maximum value that \( g(x,y) \) achieves over the region of feasibility, and determine the associated maximum profit. If your answer happens to be a decimal, then find the closest integer solutions to this problem that still satisfy the constraints.