1. [6 pts] In the process to find the reduced row-echelon form for the following system of linear equations state and execute the first 4 steps.

SHOW YOUR WORK FOR CREDIT!!!!

\[
\begin{align*}
x - y - 3z &= -8 \\
2x + y + 3z &= -1 \\
2x - 2y - 6z &= -16
\end{align*}
\]

**solution:**

\[
\begin{bmatrix}
1 & -1 & -3 & -8 \\
2 & 1 & 3 & -1 \\
2 & -2 & -6 & -16
\end{bmatrix}
\]

\((-2) r_1 + r_2 = R_2 \quad \text{and} \quad (-2) r_1 + r_3 = R_3
\]

\[
\begin{bmatrix}
1 & -1 & -3 & -8 \\
0 & 3 & 9 & 15 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\((1/3) r_2 = R_2
\]

\[
\begin{bmatrix}
1 & -1 & -3 & -8 \\
0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\(r_2 + r_1 = R_1
\]

\[
\begin{bmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 3 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

2. [6 pts] Let \(A = \begin{bmatrix} 2 & a \\ 4 & -2 \\ 6 & -1 \end{bmatrix}\) and \(B = \begin{bmatrix} 2 & 1 \\ 3 & a \end{bmatrix}\). Find \(A \cdot B\)

**solution:**

\[
A \cdot B = \begin{bmatrix}
4 + 3a & 2 + a^2 \\
2 & 4 - 2a \\
3 & 6 - a
\end{bmatrix}
\]
3. A company makes two kinds of animal food, $A$ and $B$, which contain two food supplements. It takes 2 pounds of the first supplement and 1 pound of the second to make a bag of food $A$, and 4 pounds of the first supplement and 5 pounds of the second to make a bag of food $B$. On a certain day 80 pounds of the first supplement and 70 pounds of the second is available. Suppose that the profit on each bag of food $A$ is $3.00 and the profit on each bag of food $B$ is $10.00. The company wants to maximize the profit for that particular day.

(a) [4 pts] Clearly define your variables and write the objective function, $P$, to be maximized in the above problem.

**solution:**

$x = \text{number of bags of Food type A}$

$y = \text{number of bags of Food type B}$

$P(x,y) = 3x + 10y$

(b) [4 pts] Write down all the constraints. **DO NOT SOLVE!!!**

**solution:**

$2x + 4y \leq 80$  \hspace{1cm} (restriction on available amount of first supplement)

$x + 5y \leq 70$  \hspace{1cm} (restriction on available amount of second supplement)

$x \geq 0$

$y \geq 0$