THOUGHTS ABOUT THE $\tilde{M}$-FUNCTOR, CROSSED PRODUCTS AND PARACOMPACT $C^*$-ALGEBRAS

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ABSTRACT

Many $C^*$-algebras I have met are of the form $A = BC$ where $B$ and $C$ are $C^*$-subalgebras of the multiplier algebra $M(A)$.

Example 0.1. A crossed product $A = B \rtimes_\alpha G$ with $C = C^*(G)$.

Example 0.2. $A = B \otimes C$ for two $C^*$-algebras $B, C$ and some $C^*$-tensor product.

In both examples the following $C^*$-algebra is of interest:

$$\tilde{M}(A) = \{ x \in M(A); xC + Cx \subseteq A \}.$$ 

One goal is to find out when $\tilde{M}$ is an exact functor on $C^*$-algebras. As a first step I will try to characterize $\tilde{M}(A)$ in both examples. There will be few proofs, but many conjectures.

In the first example $\tilde{M}(A)$ looks like a crossed product. In the second example there are results for $C = C_b(X)$ with $X$ paracompact. I therefore suggest that there should be a definition of paracompact $C^*$-algebras. This definition should include $C = C^*(G)$. 