1. (9) Complete the following definitions.
   (a) Let $S$ be a subset of $\mathbb{R}^n$. An element $x$ of $S$ is an interior point of $S$ if...
   (b) A sequence $(x_k)$ in $\mathbb{R}^n$ is Cauchy if...
   (c) Let $D \subseteq \mathbb{R}^n$ and $x_0 \in D$. A function $f: D \to \mathbb{R}^n$ is continuous at $x_0$ if...

2. (9) Complete the following to give meaningful true statements.
   (a) A set $S$ in $\mathbb{R}^n$ is closed if and only if...
   (b) Every bounded infinite subset of $\mathbb{R}^n$ has...
   (c) Every Cauchy sequence in $\mathbb{R}^n$...

3. (12) In each part, give an example or briefly explain why there is no such example.
   (a) A set $S \subseteq \mathbb{R}^n$ which is neither closed nor open.
   (b) A sequence in $\mathbb{R}^n$ which has no convergent subsequence.
   (c) Continuous functions $f, g: \mathbb{R}^n \to \mathbb{R}^m$ such that $f + g$ is not continuous.

4. (10) Prove, using the definition, that the function $f$ defined by
   \[ f(x, y) = x^2 + 3y \]
   is uniformly continuous on the closed ball $B = \{p \in \mathbb{R}^2 \mid |p| \leq 9\}$.

5. (15) Suppose that $f: \mathbb{R}^2 \to \mathbb{R}$ is continuous, and let
   \[ A = \{p \in \mathbb{R}^2 \mid f(p) > 0\} \quad \text{and} \quad B = \{p \in \mathbb{R}^2 \mid f(p) = 0\}. \]
   (a) Prove that $A$ is open.
   (b) Prove that $\partial A \subseteq B$.
   (c) Show by example that we can have $\partial A \neq B$.

6. (5) Suppose
   \[ f(x, y) = \frac{x^2}{x^2 + y^2} \]
   for $(x, y) \neq (0, 0)$. Prove that $\lim_{p \to 0} f(p)$ does not exist.
MAT 372 A

SAMPLE EXAM 1

February 15, 2006

INSTRUCTIONS. This is a sample exam. It is primarily meant to give you an indication of the format of the actual exam; it is more or less the same length, not necessarily the same difficulty, and probably not the same questions. The exam will be drawn from the material in Sections 1.1 through 3.3 of the text, and my lectures. For more practice, look at the extra problems from homework assignments 1–5.

You have until 10:30am to complete the 6 problems on this exam. Be sure to watch the clock and pace yourself accordingly.

No notes, books, or calculators are allowed.

Be sure to understand each problem carefully before starting work on it. Unless otherwise indicated, you may use any results from class, from the text (through Section 3.3), and from any homework you have turned in. When you use such results, be sure to make some acknowledgement. For example, “since every interval contains a rational number”, or “by the Squeeze Theorem…”

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