1. (20) Complete the following definitions:

(a) Let \( E \subseteq \mathbb{R} \) be a nonempty set. \( s \) is a supremum of \( E \) if . . .

(b) A sequence \( \{x_n\} \) is Cauchy if . . .

(c) Let \( a \in \mathbb{R} \), \( I \) be an open interval which contains \( a \), and \( f \) be a real function defined on \( I \setminus \{a\} \). Then \( f(x) \) converges to \( L \) as \( x \) approaches \( a \) if . . .

2. (20) Prove, using the definition, that \( \frac{5-n}{n^2} \to 0 \) as \( n \to \infty \).

3. (20) Prove that there exists a sequence \( \{x_n\} \subseteq \mathbb{Q} \) such that \( x_n \to \sqrt{2} \).

4. (20) Let \( \{x_n\} \) be a real sequence. Prove that

\[ x_n \to a \implies |x_n| \to |a|; \]

also prove that the reverse implication is not true.

5. (20) Let \( A \) and \( B \) be sets, and let \( f \) be a function defined on \( A \) and \( B \). Prove that

\( f(A \cap B) \subseteq f(A) \cap f(B) \), and that equality holds when \( f \) is 1–1.
Instructions:
You have 90 minutes to complete the 5 problems on the reverse of this sheet. You must
time yourself, so be sure to watch the clock and pace yourself accordingly. Write your
answers on blank paper obtained from the Testing Clerk. Write on one side of each sheet
only; put your name on each sheet and number the pages. Turn in all your work, along
with this cover sheet.

No notes, books, or calculators are allowed. Testing Center staff cannot answer your
questions. In case of confusion, your best bet is to clearly write on the exam what you are
doing, and why.

Be sure to understand each problem carefully before starting work on it, especially problems
which state “Prove, using the definition…” Be as thorough as you can in your solutions:
if you use a fact without proving it, be sure to state that you are using it. For example,
“since $\sqrt{3}$ is irrational,…”, or “by the Squeeze Theorem…”. In general, you may assume
the facts about $\mathbb{R}$ found in Chapter 1 of the text unless explicitly directed otherwise.

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