1. a. For the scalar differential equation $x' = x(1 - x^2)$ sketch a slopefield, solution graphs, and the phase-line. Identify all equilibria, and identify their stability properties.
   b. Sketch a bifurcation diagram for the family of scalar DEs $x' = x(a - x^2)$ for $a \in \mathbb{R}$.
   c. Outline an argument why the differential equation $x' = x(1 - x^2) + \sin(t)$ has a periodic solution (or does not have any periodic solutions).

2. For each of the following give an example of an initial value problem (IVP) $x' = f(t, x)$, $x(t_0) = x_0$ with the indicated property. Briefly justify your answers.
   a. The IVP does not have any solution.
   b. The IVP does have more than one solution (on the same domain).
   c. $f$ is defined on $\mathbb{R}^2$, but no solution is defined on all of $\mathbb{R}$.

3. a. For each matrix $A$ below classify the phase portrait of $x' = Ax$. Justify your answer.
   b. For ONE of the matrices $A$ above find a matrix $T$ such that $T^{-1}AT$ is in canonical form.
      
      (i) $\begin{pmatrix} 6 & 7 \\ 8 & 9 \end{pmatrix}$  
      (ii) $\begin{pmatrix} 7 & 6 \\ 8 & 9 \end{pmatrix}$  
      (iii) $\begin{pmatrix} 6 & 9 \\ -8 & -7 \end{pmatrix}$  
      (iv) $\begin{pmatrix} 6 & 9 \\ -6 & -9 \end{pmatrix}$

4. a. Find all eigenvalues and all eigenvectors of the matrix $A$ given below on the right.
   b. Use the rank of the matrix $N = A - I$ to identify the Jordan normal form $J$ of $A$.
   c. Compute the matrix exponentials $e^{tN}$, $e^{tJ}$, and $e^{tA}$.
   d. Solve the initial value problem $x' = Ax + b(t)$, $x(0) = (0, 1, 0)^T$.

   $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \end{pmatrix}$, $b(t) = \begin{pmatrix} 0 \\ e^{-t} \\ 0 \end{pmatrix}$

   **Bonus**: Find a matrix $Q$ such that $AQ = QJ$ and outline how to use this to compute $e^{tA}$.
   **Bonus**: Outline an argument why for every matrix $A$ its exponential $e^A$ is invertible.

5. a. Write the initial value problem $x'' + \sin x = 0$, $x(0) = 0$, $x'(0) = 1$ as a first order system.
   b. Calculate the first three successive approximations using Picard iteration