1. Suppose $X$ and $Y$ are sets, and $f: X \rightarrow Y$. Prove or provide a counterexample:
   a. For every $A \subseteq X$, $(f(A))^C = f(A^C)$ (where $A^C = X \setminus A$).
   b. For every $B \subseteq Y$, $(f^{-1}(B))^C = f^{-1}(B^C)$ (where $B^C = Y \setminus B$).

2. Suppose $B_1$ and $B_2$ are bases for topologies $T_1$ and $T_2$ on the same space $X$.
   a. Prove that if $B_1 \subseteq B_2$ then $T_1 \subseteq T_2$.
   b. Is the converse true? Justify your answer.
   c. If $A \subseteq X$ and $T_2$ is finer than $T_1$ what can you say about how the closures $c\ell_1(A)$ and $c\ell_2(A)$ of $A$ in either topology compare? Prove that you are correct.

3. On $\mathbb{R}$ consider the collections of intervals $B_s = \{(a, b): a < b\}$ and $B_\ell = \{[a, b): a < b\}$.
   a. Verify that $B_\ell$ is a basis for a topology on $\mathbb{R}$.
   b. Let $T_s$ and $T_\ell$ denote the topologies generated by $B_s$ and $B_\ell$, respectively.
      Decide whether $T_s$ is finer or coarser than $T_\ell$, or neither – and prove that you are correct.

4. Suppose $X$ and $Y$ are topological spaces, $f: X \rightarrow Y$ is continuous.
   a. Prove or give a counterexample: If $X$ is Hausdorff then $Y$ is Hausdorff.
   b. Prove or give a counterexample: If $Y$ is Hausdorff and $f$ is onto, then $X$ is Hausdorff.
   c. Prove or give a counterexample: If $Y$ is Hausdorff and $f$ is one-to-one, then $X$ is Hausdorff.

5. Suppose $X$ and $Y$ are metric spaces and $f: X \rightarrow Y$.
   Prove that if $f$ satisfies the $\varepsilon\delta$-characterization of continuity in metric spaces then $f$ is also continuous according to the open-set definition in general topological spaces.

6. Suppose that $X$ is a metric space $X$, $z \in X$, $A \subseteq X$, and $\overline{A}$ is the closure of $A$.
   a. Show that if $z \in \overline{A}$ then there exists a sequence $a = (a_n)_{n \in \mathbb{Z}^+} \subseteq A$ that converges to $z$.
   b. Give an explicit counterexample to show that without the assumption that $X$ is metric there need not exist such a sequence.

7. For each of the given functions $f, g: \mathbb{R} \rightarrow \mathbb{R}^\omega$ decide whether it is continuous when
   (i) $\mathbb{R}^\omega$ is equipped with the product topology, and when (ii) $\mathbb{R}^\omega$ is equipped with the box topology.
   a. $f(t) = (t, 2t, 4t, 8t, 16t, \ldots)$
   b. $g(t) = (t, t/2, t/4, t/8, t/16, \ldots)$