1. a. Define upper bound and least upper bound.
   b. State the least upper bound axiom (or supremum axiom).
   c. Define open ball and open set.

2. Using only the field axioms, prove the following (if impossible explain why):
   a. \( \forall x, y \in \mathbb{R}, \text{if } x \cdot y = 0 \text{ then } x = 0 \text{ or } y = 0, \) and
   b. \( \forall x, y \in \mathbb{R}, \text{if } x \neq 0 \text{ and } y \neq 0 \text{ then } (xy)^{-1} \text{ exists, and } (xy)^{-1} = x^{-1}y^{-1}. \)

3. a. Show that if a set \( S \subseteq \mathbb{R} \) has a least upper bound, then the least upper bound is unique.
   b. Give a detailed outline of a proof that there exists a real number \( z \) such that \( z^2 = 3. \)

4. a. Prove that the intersection of two open sets is open.
   b. Prove that the union of any collection of open sets is open.
   c. Give an example of a countable collection of open intervals whose intersection is the closed interval \([0, 1]\).

5. Suppose that \((a_n)_{n=1}^\infty\) and \((b_n)_{n=1}^\infty\) are converging sequences of real numbers. Working from the definition of “convergence” prove that the sequence \((3a_n - 2b_n)_{n=1}^\infty\) converges.

6. Suppose that \((a_n)_{n=1}^\infty\) and \((b_n)_{n=1}^\infty\) are sequences of real numbers that converge to real numbers \( L \) and \( M \), respectively.
   a. Prove that if for all \( n \in \mathbb{Z}^+ \) \( a_n < b_n \), then \( L \leq M. \)
   b. Give an explicit counterexample that shows that (under the same hypotheses as above) the conclusion \( L \leq M \) cannot be replaced by \( L < M. \)
   c. Conversely suppose that \((a_n)_{n=1}^\infty\) and \((b_n)_{n=1}^\infty\) are sequences of real numbers that converge to real numbers \( L \) and \( M \), respectively. What can you say (without proof) about the relationship between \( a_n \) and \( b_n \) if you know only that \( L < M? \)