1. a. State the definition of Cauchy sequence.
   b. Briefly summarize the difference between limit and limit point.
   c. State the definition of compact.

   **Bonus.** Give several different arguments that show that \( \mathbb{Q} \cap [0, 10] \) is not compact.

2. For each of the following subsets of \( \mathbb{R} \) find (without proof) the set of all limit points.
   a. \((0, 1]\)
   b. \(\mathbb{Q}\)
   c. \(\{(-1)^n \cdot \frac{n}{n+1} : n \in \mathbb{Z}^+ \}\)
   d. \(\{2^{-n} : n \in \mathbb{Z}\}\)

3.a. Suppose that \( K, a \in \mathbb{R} \), and \( f: \mathbb{R} \to \mathbb{R} \) is such that for all \( x \neq a \), \( f(x) > K \). Assuming that \( \lim_{x \to a} f(x) \) exists use the \( \varepsilon-\delta \)-definition of the limit to show that \( \lim_{x \to a} f(x) \geq K \).
   b. Give an example that shows that it need not be true that \( \lim_{x \to a} f(x) > K \).

4. a. Prove that every Cauchy sequence (in a metric space) is bounded.
   b. Outline the key steps of an argument that shows that \( \mathbb{R} \) is complete.

5. a. Suppose that \( f: X \to Y \) is a uniformly continuous function between metric spaces, and \((a_n)_{n=1}^\infty \) is a Cauchy sequence in \( X \). Prove that \((f(a_n))_{n=1}^\infty \) is a Cauchy sequence.
   b. Give an example that shows that the assumption of uniform (continuity) cannot be omitted.

6. Prove one of the following two theorems.
   If \( K \) is compact, \( Y \) a metric space, and \( f: K \to Y \) continuous, then \( f(K) \) is compact.
   If \( g: X \to Y \) and \( f: Y \to Z \) are continuous functions between metric spaces then \( f \circ g \) is continuous.