1. State (complete) the “standard” definitions for:
   a. An infinite sequence \((a_n)_{n \in \mathbb{Z}^+}\) is a Cauchy sequence if . . .
   
   b. Suppose \(S \subseteq \mathbb{R}\) and \(z \in \mathbb{R}\). Then \(z\) is an accumulation point of \(S\) if . . .
   
   c. Suppose \(D \subseteq \mathbb{R}\) and \(f: D \mapsto \mathbb{R}\). The function \(f\) is uniformly continuous if . . .

2. a. State the Heine-Borel theorem.

   b. State two theorems which guarantee that a sequence \(a = (a_n)_{n \in \mathbb{Z}^+}\) converges,

3. For each of the following sets \(S_i \subseteq \mathbb{R}\) find the set \(S'_i\) of all accumulation points of \(S_i\) and the closure \(\overline{S_i}\). (No proofs required).
   
   a. \(S_1 = (0, \infty)\)
   
   b. \(S_2 = \mathbb{Q}\) (rational numbers)
   
   c. \(S_3 = \mathbb{Z}\) (integers)
   
   d. \(S_4 = \{2^n : n \in \mathbb{Z}\}\). (Caution: \(n\) may be negative or positive.)

4. Suppose \(D, E \subseteq \mathbb{R}, g: D \mapsto E\) is continuous at \(x_0 \in D\) and \(f: E \mapsto \mathbb{R}\) is continuous at \(y_0 = g(x_0)\). Prove that \(F = f \circ g\) is continuous at \(x_0\).

5. Let \(D = \mathbb{R} \setminus \{0\}\) and \(f: D \mapsto \mathbb{R}\) be defined by \(f(x) = \frac{1}{x}\). 
   
   a. Is \(f\) continuous? \textbf{Prove} that your answer is correct!
   
   b. Is \(f\) uniformly continuous? \textbf{Prove} that your answer is correct!
   
   (In either part you may use any theorem proved in class or in the homework).

6. Let \(Q\) be the set of rational numbers and let \(f: Q \mapsto \mathbb{R}\) be defined by \(f(x) = \frac{1}{q}\) if \(x = \frac{p}{q}\) with \(p, q \in \mathbb{Z}, q > 0\) and \(\gcd(p, q) = 1\).

   For each of \(x_1 = \frac{2}{3}\) and \(x_2 = \sqrt{2}\) either prove that \(lim_{x \to x_i} f(x)\) does not exist, or calculate the limit and prove that it is indeed the limit.