1. State precise definitions for the following

a. A function $f$ is increasing on an interval $[a, b]$ if . . .

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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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b. A function $f$ is integrable over an interval $[a, b]$ if . . .

c. A sequence of functions $(f_n)_{n=1}^\infty$ converges pointwise if . . .

b. A sequence of functions $(f_n)_{n=1}^\infty$ converges uniformly if . . .

2.a. State both forms of the Fundamental Theorem of Calculus.

b. Suppose $f: \mathbb{R} \mapsto (0, \infty)$ is continuous and integrable over every finite interval $[a, b]$. Show that the function $F: \mathbb{R} \mapsto \mathbb{R}$ defined by $F(x) = \int_0^x f(t) \, dt$ is strictly increasing.

3. Suppose that $a < b$ are real numbers, and $f, g: [a, b] \mapsto \mathbb{R}$ are integrable functions such that for all $x \in [a, b]$, $f(x) \leq g(x)$.

a. Working directly from the definition of the Riemann integral, show that $\int_a^b f \leq \int_a^b g$.

b. Give an example of two functions $f$ and $g$ such that for all $x \in [a, b]$, $f(x) \leq g(x)$, and $f$ and $g$ that are not equal to each other, but $\int_a^b f = \int_a^b g$.

4. For $n \in \mathbb{Z}^+$ consider functions $f_n: [0, 1] \mapsto \mathbb{R}$, defined by $f_n(x) = x^n$.

a. Show that the sequence $(f_n)_{n=1}^\infty$ converges pointwise.

b. Show that the sequence $(f_n)_{n=1}^\infty$ does not converge uniformly.

**Bonus.** Show that the limit of a uniformly convergent sequence of continuous functions is continuous.