1. State precise definitions for the following
   a. A function $f$ is increasing on an interval $(a, b)$ if . . .

   b. A function $f$ is differentiable on an interval $(a, b)$ if . . .

   c. A function $f$ is integrable over an interval $[a, b]$ if . . .

2. a. State the Extreme Value Theorem.

   b. State the Mean Value Theorem.

3. Working directly from the definition
   a. show that every differentiable function is continuous, and

   b. show that a differentiable function $f$ is increasing on an interval $(a, b)$
      if and only if $f' \geq 0$ on $(a, b)$.

4.a. Suppose that $a < b$ are real numbers. Working directly from the definition
      show that if a function $f: [a, b] \mapsto [0, \infty)$ is integrable then $\int_a^b f \geq 0$.

   b. Given an example of an integrable function $f: [a, b] \mapsto [0, \infty)$
      such that $\int_a^b f = 0$, but $f \neq 0$.

   **Bonus.** Show that if $f: [a, b] \mapsto [0, \infty)$ is continuous (and hence integrable),
   and $f \neq 0$ then $\int_a^b f > 0$. 