1. a. Give a precise definition of \textit{least upper bound}. 
   
   b. State the \textit{least upper bound axiom} (or \textit{supremum axiom}). 
   
   c. State the definition for \textit{convergence of a sequence}. 
   
   \textbf{Bonus.} State the definition of \textit{closed set}. 

2. a. Using only the field axioms, prove \textbf{ONE} of the following: 
   \begin{itemize} 
   \item (i) \( \forall x \in \mathbb{R}, \ 0 \cdot x = 0 \), 
   \item (ii) \( \forall x \in \mathbb{R}, \ (-1) \cdot x = -x \). 
   \end{itemize} 
   If impossible explain why. 
   
   b. Is it possible, using only the field axioms, to prove that \( \frac{1}{2} \) exists? \textbf{Explain} why (not). 

3. a. Prove that the sequence \((a_n)_{n=1}^{\infty}\) defined by 
   \[ a_n = \begin{cases} 
   1 & \text{if } n \text{ is odd} \\
   n & \text{if } n \text{ is even} 
   \end{cases} \] 
   \( \) does not converge. 
   
   b. If possible find a converging subsequence of \((a_n)_{n=1}^{\infty}\) and \textbf{prove} that this converges. 
   If impossible \textbf{explain} why. 

4. Suppose that \((a_n)_{n=1}^{\infty}\) and \((b_n)_{n=1}^{\infty}\) are converging sequences of real numbers. 
   Prove that the sequence \((a_n - b_n)_{n=1}^{\infty}\) converges. 

5. Outline the main steps of a proof that there exists a number \( x \in \mathbb{R} \) such that \( x^2 = 2 \). 

6. a. What can you say about unions, intersections, and complements of open sets? 
   State – without proof – the strongest statements that you know to be true. 
   
   b. Give an example of a countable collection of closed intervals whose union 
   is the open interval \((0, \infty)\). 
   
   \textbf{Bonus.} Prove that the intersection of two open sets is open. (Work from the definition).