1. Suppose that \( f: \mathbb{N} \to \mathcal{P}(\mathbb{N}) \)
   a. Prove that \( f \) is not onto.
   b. Construct an explicit example of such \( f \) that is one-to-one.

2. Prove that there does not exist a rational number \( x \) such that \( x^2 = 3 \).
   (You may use, without having to prove it, that for any \( n \in \mathbb{Z}^+ \), if \( 3 | n^2 \) then \( 3 | n \).)

3. Let \( \sim \) be an equivalence relation on a set \( A \).
   a. Suppose \( a \in A \). Give a precise definition of the equivalence class \( [a] \) of \( a \).
   b. Show that if \( a, b \in A \) are such that \( [a] \cap [b] \neq \emptyset \) then \( a \sim b \).

4.a. Suppose \( k, m, n \) are integers such that \( k|m \) and \( m|n \). Prove that \( k|n \).
   b. For every positive integer \( n \in \mathbb{Z}^+ \) let \( D(n) = \{ k > 1: k|n \} \).
      Show that for every \( n > 1 \), \( D(n) \neq \emptyset \) and the smallest element in \( D(n) \) is prime.

5.a. Show that if \( m, n, q, r \in \mathbb{N}, m \neq 0 \), and \( n = mq + r \) then \( \gcd(m, n) = \gcd(m, r) \).
   b. Suppose \( m, n \in \mathbb{N} \), not both zero. Prove that there exist \( s, t \in \mathbb{Z} \) such that \( ms + nt = \gcd(m, n) \).

6. Find an integer \( x \) such that \( x \cdot 17 \equiv 1 \mod 4003 \).

7. Consider the function \( f: \mathbb{N} \to \mathcal{F}(\mathbb{N}) \) (the set of finite subsets of \( \mathbb{N} \)) defined recursively by \( f(0) = \emptyset \),
   and, if \( n \geq 1 \), then \( f(n) = \{ k \} \cup f(n - 2^k) \) where \( k \in \mathbb{N} \) is such that \( 2^k \leq n < 2^{k+1} \).
   a. Find \( f(37) \).
   b. Find \( x \in \mathbb{N} \) such that \( f(x) = \{0, 1, 6\} \).

Bonus:
   c. Prove by induction on \( n \): For every \( n \in \mathbb{N} \), if \( S \in \mathcal{F}(\mathbb{N}) \) and \( m \leq n \) for all \( m \in S \), then there
      exists \( x \in \mathbb{N} \) such that \( f(x) = S \).
      Explain why this implies that \( f \) is onto.