Objectives: Practice translating statements from “English” into symbols (with quantifiers and predicates) and vice versa. Practice symbolic rules for negation. Practice rephrasing statements in different ways, and recognizing when two such statements are logically equivalent. Very important: Enjoy and have fun!

Definition: \( L(x, y): \) “\( x \) loves \( y \).”

There are eight different ways (orders) to quantify \( L(x, y) \) with \( \forall, \exists, x \leq y \) in a meaningful way. Six of them generally have different meanings. What are they?

For each of these there are 3 different places to insert a negation, and three different ways to insert two negations, and one way to insert no, or three negations each. (This assumes that no two negation are directly following each other). These negations can be generally be rearranged – following the usual rules of predicate calculus – so that several many of these statements are equivalent, e.g. the following are all logically equivalent \( \neg \forall x \exists y L(x, y), \exists x \neg \exists y L(x, y), \exists x \forall y \neg L(x, y), \) and \( \neg \forall x \neg \forall y \neg L(x, y) \).

Thus we obtain 12 logically inequivalent statements that are written in 48 different forms. Some are easier to translate into English, than others. Try yourself. The following examples are a try – likely with some mistakes.

Take a brief look at the table below – but then put it away. Then try to organize all these different statements in a nice way, verify which are equivalent, and which are negations of each other. Finally, for each group of four equivalent statements formulate reasonably nice sentences in plain English – rewrite in several different ways.

The negate your sentences in English, and see whether the negated sentences match the ones found by translating the negated symbolic sentences.

I hope you will have some heated discussions with your friends – and please let me know of the mistakes that are in the table, and of all other disagreements.

| \( \exists x \exists y \ L(x, y) \) | \( \neg \forall x \forall y \neg L(x, y) \) | “Somebody loves somebody.”
|-------------------------------|---------------------------------| “Not everybody loves nobody.”
| \( \forall x \forall y \ L(x, y) \) | \( \neg \exists x \exists y \neg L(x, y) \) | “Everybody loves everybody.”
| | | “Everybody loves everybody.”
| | | “Not everybody does not love somebody.” ??
| \( \exists x \forall y \ L(x, y) \) | \( \neg \forall x \forall y \neg L(x, y) \) | “Somebody loves everybody.”
| | | “Not everybody does not love somebody.” ??
| \( \forall x \exists y \ L(x, y) \) | \( \neg \exists x \forall y \neg L(x, y) \) | “Somebody loves everybody.”
| \( \neg \exists x \exists y \ L(x, y) \) | \( \forall x \forall y \neg L(x, y) \) | “Somebody loves everybody.”
| | | “Nobody loves anybody.” ??
| | | “Everybody loves somebody.”
| | | “Not everybody loves nobody.” ??
| \( \forall x \exists y \ L(x, y) \) | \( \forall x \exists y \neg L(x, y) \) | “Everybody is loved by somebody.”
| | | “Everybody loves nobody.”
| | | “Somebody does not love everybody.” ??
| | | “Not everybody loves everybody.” ??

This table is far from complete – many missing cells, and arbitrarily left out equivalent statements such as: \( \forall y \exists x \ L(x, y) \) “Everybody is loved by somebody.”. I expect that there are several mistakes – some type-ohs, some genuine errors. Use it carefully as a reference to compare with your own work.