MAT 300 Mathematical Structures

Exercises for immediate successors etc.

1. Consider the positive integers $\mathbb{Z}^+$ with the usual order relation.
   Which number(s) has(ve) no immediate successor?
   Which number(s) has(ve) no immediate predecessor?

2. Consider the rational numbers $\mathbb{Q}$ with the usual order relation.
   Which number(s) has(ve) no immediate successor?
   Which number(s) has(ve) no immediate predecessor?

3. Consider the real numbers $\mathbb{R}$ with the usual order relation.
   Which number(s) has(ve) no immediate successor?
   Which number(s) has(ve) no immediate predecessor?
   What is the immediate predecessor of 1 (if it exists)?
   What is the immediate successor of 0 . . . (if it exists)?

4. Let $S = \mathbb{Z}^+ \setminus \{1\}$ with a partial order defined by $m \leq n$ is there is $q \in S$ such that $mq = n$.
   Which number(s) in $S$ has(ve) no immediate successor?
   Which number(s) $S$ has(ve) no immediate predecessor?
   Find all immediate predecessors of 12 (if any exist).
   Find all immediate successors of 12 (if any exist).
   Find all immediate predecessors of 16 (if any exist).
   Find all immediate successors of 16 (if any exist).
   Which number(s) in $S$ has(ve) exactly one immediate predecessor?
   Which number(s) in $S$ has(ve) exactly one immediate successor?

5. Consider the set $W$ of all words (not necessarily in English) that can be formed from the (upper-case, only) letters \{A, B, C, . . . , Z\} under the usual dictionary order.
   Which word(s) in $W$ has(ve) no immediate successor?
   Which word(s) in $W$ has(ve) no immediate predecessor?
   Find all immediate predecessors of the word PROOF (if any exist).
   Find all immediate successors of the word PROOF (if any exist).
   Find all immediate predecessors of the word BETA (if any exist)
   Find all immediate successors of the word BETA (if any exist).

6. Let $S = \mathbb{Z}_0^+ \times \mathbb{Z}_0^+$ of pairs of nonnegative integers with a partial order defined by $(a, b) \leq (c, d)$ if either $a < c$ or $a = c$ and $b \leq d$ (usual order on $\mathbb{Z}$).
   Which pair(s) in $S$ has(ve) no immediate successor?
   Which pair(s) $S$ has(ve) no immediate predecessor?
   Find all immediate predecessors of (3, 7) (if any exist).
   Find all immediate successors of (3, 7) (if any exist).
   Find all immediate predecessors of (3, 0) (if any exist).
   Find all immediate successors of (3, 0) (if any exist).

7. Let $S = \mathbb{R}_0^+ \times \mathbb{R}_0^+$ of pairs of real numbers with a partial order defined by $(a, b) \leq (c, d)$ if either $a < c$ or $a = c$ and $b \leq d$ (usual order on $\mathbb{R}$).
   Which pair(s) in $S$ has(ve) no immediate successor?
   Which pair(s) $S$ has(ve) no immediate predecessor?
   Find all immediate predecessors of (3, 7) (if any exist).
   Find all immediate successors of (3, 7) (if any exist).
   Find all immediate predecessors of (3, 0) (if any exist).
   Find all immediate successors of (3, 0) (if any exist).