1. Suppose we understand the free variable \( z \) to refer to (a) books, (b) automobiles, and (c) pencils. For each context,
   - Give an example of a predicate \( A(z) \) for which “For all \( z, A(z) \)” is a true statement.
   - Give an example of a predicate \( B(z) \) for which “For all \( z, B(z) \)” is false but “There exists \( z \) such that \( B(z) \)” is true.

2. Is it possible to have a predicate \( T(x) \) such that “For all \( x, T(x) \)” is true, but “There exists some \( x \) such that \( T(x) \)” is false? Justify your answer.

3. Consider the statements
   
   \[ \begin{align*}
   P & := “Dogs eat meat.” \\
   Q & := “Rome is in Italy.” \\
   R & := “Chocolate prevents cavities.” \\
   S & := “The moon is made of green cheese.”
   \end{align*} \]

   Determine whether each of the following is true or false.
   
   (a) If \( P \), then \( Q \). 
   (b) If \( P \), then \( R \). 
   (c) If \( R \), then \( S \).
   (d) If \( S \), then \( Q \). 
   (e) If \( Q \), then \( S \).

4. With apologies to Sidney Harris for trodding on his terrific cartoon (shown in Figure 1.1), I’d like to play a little with the dog’s statement. Consider the assertions made by the dog:
   
   \[ \begin{align*}
   A & := “All cats have four legs.” \\
   B & := “I have four legs.” \\
   C & := “I am a cat.”
   \end{align*} \]

   (Are these assertions statements or predicates? Explain.)

   The dog’s statement is of the form “if \( A \) and \( B \), then \( C \).”

   (a) Construct a truth table for the statement “If \( A \) and \( B \), then \( C \).”

   (b) Now consider the actual truth values of the assertions made by the dog. Cross out the lines of the truth table that don’t apply in this particular instance. What do you see?

5. This problem refers to the equivalence discussed in Example 1.7.6.

   (a) Using your intuition about implication, explain why it makes sense to say that

   \[ If \ A \ is \ true, \ then \ either \ B \ is \ true \ or \ C \ is \ true \]

   means the same thing as

   \[ If \ A \ is \ true \ and \ B \ is \ false, \ then \ C \ is \ true. \]

   (b) Go back to the “thought experiment” in Section 1.1. Find a statement that is written in the form “If \( A \), then \( B \) or \( C \).” Find two equivalent rephrasings of the statement. (Did you intuit these rephrasings when you worked with the problem during the thought experiment?)

   (c) Construct a truth table to show that \((A \implies (B \lor C)) \) cannot be rephrased as \((A \implies B) \lor (A \implies C)\). Using the statement you discussed in part (b) as an example, explain why.

6. Consider the statement “Marlene has brown hair.” When asked to negate this statement, some students are apt to say, “Marlene has blond hair.” Explain why this is incorrect. (Hint: There