It is your responsibility to demonstrate that you have mastered the material of the class. Use the computers to validate your solutions, and to expedite basic calculations. **Solution formulas alone**, such as provided by MAPLE do NOT earn MUCH credit. Much more important are sound justifications and explanations, and convincing demonstrations that you have mastered the concepts and solution **methods** from this class. Scratch work, such as formulas scattered across the page without clear logical order, will be ignored and earn zero credit. 

Throughout, $u(t)$ denotes the Heaviside function 

$$u(t) = \begin{cases} 
0 & \text{if } t < 0 \\
1 & \text{if } t \geq 0 
\end{cases}$$

1. Demonstrate how to use Laplace transforms to solve the initial value problem 

$$y'' + 7y' + 12y = 0, \quad y(0) = 2, \quad y'(0) = -1.$$ 

2. **a.** Assuming that it is legitimate to differentiate under the integral sign, show that 

$$\mathcal{L}\{f(t)\}(s) = F(s) \text{ then } \mathcal{L}\{tf(t)\}(s) = -\frac{d}{ds}F(s).$$ 

What is $\mathcal{L}\{t^4f(t)\}(s)$? 

**b.** Use part **a.** to derive the Laplace transform of $t^4e^{10t}$. 

**c.** Use “real” calculus II or complex variables, to derive the table entry for the Laplace transform of cost $t$. 

3. **a.** Find possible formulas using Heaviside functions for the functions sketched on the right. 

**b.** Find the transform of each function (**use any method**, table or MAPLE are OK). 

4. Find the inverse Laplace transform of 

$$\frac{24e^{-10s}}{s^5} + \frac{1}{s} \cdot \frac{8}{s-1}.$$ 

Demonstrate your ability to manipulate the formula to make it ready for table look-up. 

**Bonus** Use the identity $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$ to find the inverse Laplace transform of $\frac{1}{1+e^{-x}}$. Sketch the graph of your answer. 

5. For $b = 1$ solve the initial value problem. 

$$y' + by = bu(t - 4) - bu(t - 8) + bu(t - 12) - bu(t - 16), \quad y(0) = 1.$$ 

Plot the solution together with the forcing term, and explain in physical terms what you see. 

*If you showed lots of detail in problem 1, you need not repeat the steps here. Manage your time.*

**Bonus.** Describe in words how the solution changes when $b$ is smaller than 1, e.g. $b = \frac{1}{2}$, and when $b$ is larger than 1, e.g. $b = 3$, $b = 10$, or $b = 100$?