It is your responsibility to demonstrate that you have mastered the material of the class. Use the computers to validate your solutions, and to expedite basic calculations. **Solution formulas alone**, such as provided by MAPLE do NOT earn MUCH credit. Much more important are sound justifications and explanations, and convincing demonstrations that you have mastered the concepts and solution methods from this class. Scratch work, such as formulas scattered across the page without clear logical order, will be ignored and earn zero credit.

1. **a.** Solve the initial value problem \( y'' + 6y' + 5y = 0, \ y(0) = -1, \ y'(0) = 13. \)
   
   **b.** Sketch the graph of the solution by hand. Label your axes (and define the scales).
   
   **c.** Calculate the **maximal overshoot** \( y_{\text{max}} = \max \{ y(t) : t > 0 \} \).
   
   The final answer should be a four-digit accuracy decimal number. But intermediate symbolic/algebraic work earns more respect and credit than purely numeric/graphical work.
   
   **d.** Replacing the **damping parameter** 4 in the above differential equation by a free parameter \( \gamma \), determine for which values of \( \gamma \) the system will exhibit (i) overdamped, (ii) critically damped, (iii) underdamped, (iv) undamped, and (v) unstable oscillations?

2. **a.** Find the general solution of the differential equation \( y'' - 6y' + 9y = \cos mt \) where \( m > 0 \) is any positive integer.
   
   **b.** Find the general solution of the differential equation \( y'' - 6y' + 9y = e^{-mt} \) where \( m > 0 \) is any positive integer. **Be careful:** Some values of \( m \) may need special treatment!
   
   **c.** Find the solution of the initial value problem
   
   \[ y'' - 6y' + 9y = 16e^{-t} + 20e^{3t} + 18 \cos 3t, \quad y(0) = 1, \quad y'(0) = 0. \]

3. **a.** Find the general solution of the differential equation \( y'' + y' + 4y = e^{i\omega t} \) where \( \omega > 0 \) is any positive real number.
   
   If you prefer, you may (for the same credit) replace the forcing term \( e^{i\omega t} \) by \( \cos \omega t \).
   
   **b.** Sketch the graph of the **amplitude** of the **steady state solution** as a function of the frequency \( \omega \).
   
   **c.** Calculate the **resonance frequency** \( \omega_0 \) which maximizes the amplitude of the steady state solution.

   To maximize credit, demonstrate how to obtain this frequency from basic principles, rather than just using a memorized formula.

**Bonus.** Explain how the value of the resonance frequency depends (or why it does not depend) on the damping parameter, i.e. consider the resonance frequency \( \omega_0 \) as a function of the parameter \( \gamma \) in the differential equation \( y'' + \gamma y' + 4y = e^{i\omega t} \).

**Suggestion:** Sketch the graph of \( \omega_0 \) as a function of \( \gamma \) and explain.