(1) Find the inflection points of \( f(x) = e^{-x^2} \).

(2) Find the area of the region between the graphs of \( y = \frac{1}{x}, \ y = \sqrt{x}, \) and \( x = 4 \).

(3) What is wrong in the following calculation:

\[
\int_{-2}^{1} \frac{1}{x^2} \, dx = \left[ x^{-2} \right]_{-2}^{1} = -1 - (-1) \left( \frac{1}{-2} \right) = -\frac{3}{2}.
\]

(4) Expand \( f(x) = x \sin x \) in a Taylor series about \( x = 0 \).

(5) Mark the following quantities (points, slopes, areas, lengths,...) on the graph of \( y = f(x) \).

a) \( f(b) - f(a) \)
b) \( \frac{f(b) - f(a)}{b - a} \)
c) \( f'(a) \)
d) \( F(b) - F(a) \) where \( F' = f \)
e) \( F'(b) \)
1. In each of the following sketch the direction of the vector $\vec{v} \times \vec{w}$.

![Diagrams showing vector directions](image)

2. If $\vec{v} = (a, b, 0) = ai + bj$ and $\vec{w} = (c, d, 0) = ci + dj$

   find $\vec{v} \cdot \vec{w} =$

   and $\vec{v} \times \vec{w} =$

3. Find the angle at which two diagonals intersect in a rectangular box with square base, that is twice as high as wide.

Do all pairs of diagonals intersect at the same angles? Explain!
Consider a cube with side-length one, conveniently placed in a coordinate system as shown.

a) Find the components of the vectors
\[
\vec{AC} = \vec{v} =
\]
and
\[
\vec{AH} = \vec{w} =
\]

b) Find the angle between the two vectors \( \vec{v} \) and \( \vec{w} \).

c) Find the area of the triangle \( \triangle ACH \).

d) Find a vector from some corner of the cube to some other corner that points in the same direction as the vector \( \vec{v} \times \vec{w} \).

e) Write down the equation of the plane through \( \triangle ACH \).
1. For \( \vec{v} \) and \( \vec{w} \) as shown on the right, sketch the direction of \( \vec{v} \times \vec{w} \).

2. If \( \vec{v} \) has length 2, lies in the xy-plane and makes an angle of \( \frac{\pi}{4} \) with each the positive x- and y-direction, and \( \vec{w} \) has length 3 and points in the positive z-direction, find
   a) the direction of \( \vec{v} \times \vec{w} \)
   b) the magnitude of \( \vec{v} \times \vec{w} \) (justify your answer!)
   c) the dot product \( \vec{v} \cdot \vec{w} \)

3. Calculate \( (2\hat{i} + 3\hat{j}) \times (5\hat{j} - 7\hat{k}) = \)

   or as column-vectors: \( \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 5 \\ -7 \end{pmatrix} = \)
a.) Find the area of the triangle ABC with corners at \((1,0,0), (0,1,0)\) and \((0,0,1)\).

b.) Find a vector \(\mathbf{N}\) that is perpendicular to the triangle.

c.) Write down the equation of the plane that contains the triangle ABC.

d.) Find the number \(t\) for which the point \(R(t) = (t, t, t)\) lies on the plane.

e.) Find the distance between the point \(R\) and the origin.

for extra credit: Explain why \(R\) is the point on the plane closest to the origin. (Use reverse side.)
MAT 272  Calculus & Anal. Geometry III
Quiz 4  F  Oct 1, 93

The picture shows a piece of the graph of a function
\( z = f(x, y) \) (the curved surface ADLG), and a piece
of the tangent plane to the graph at the point A (ACIF).

In terms of \( a, b, h, u, u_2, f \) etc. find expressions for the following

1. The \( z \)-coordinate of \( A \):
2. The length of the line segment \( HL \):
3. The slope of the secant line through the points \( A \) and \( G \):

Which line segments in the picture have the slopes

4. \( \frac{\partial f}{\partial y} (a + hu_1, b) \)
5. \( \frac{f(a + hu_1, b + hu_2) - f(a, b)}{h} \)

For extra credit:
6. What is the slope of the tangent line \( AI \)?
7. What is the area of the parallelogram ACIF?
1. If \( f(x) = x^2 \), find \( df \) at \( x_0 = 10, \ dx = 1 \).
\[ \text{If } y = x^2 \text{, find } dy \text{ at } a = 10, \ dx = 1. \]

2. If \( s \) is a function of \( a \) and \( t \), \( s = \frac{1}{2}at^2 \)
find the differential \( ds \).

Suppose \( a_0 = 3 \), \( t_0 = 10 \), and \( dt = -1 \).

How do you have to choose \( da \) in order to guarantee \( ds = 0.2 \)?
MAT 272 Calculus & Analytic Geometry III
Quiz 6 Friday Oct. 10, 93

If \( x = t \), \( y = 1 - t^2 \) and \( z = \frac{1}{1+x^2+y^2} \), find:

\( 1 \) \( z \) as a function of \( t \): \( z = \)

\( 2 \)

\( \frac{dx}{dt} = \), \( \frac{dy}{dt} = \), \( \frac{dz}{dx} = \)

\( 3 \)

Write down a general formula (chain rule), that relates the derivatives \( \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dx}, \frac{dz}{dy} \).
Using a Riemann-sum (e.g. with $N = 4$) obtain a rough estimate for the definite integral

$$\int_0^2 3^{2x} \, dx \approx$$

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- No partial credit on this problem ---

Suppose $y = f(x)$ is increasing and concave up, and $y = g(x)$ is increasing and concave down.

Using Left-Riemann sums will one get

□ an overestimate or □ an underestimate for $\int_a^b f(x) \, dx$,

and □ an overestimate or □ an underestimate for $\int_a^b g(x) \, dx$. 

---
1. Label the picture below with Cartesian, cylindrical, and spherical coordinates: \( x, y, z, r, \theta, \phi \). (If you prefer draw your own picture(s) instead.)

2. List the formulas for converting coordinates:

\[
\begin{align*}
\text{Cartesian } & \leftrightarrow \text{ Cylindrical} \\
x &= \\
y &= \\
z &= \\
\text{Cylindrical } & \leftrightarrow \text{ Cartesian} \\
r &= \\
\theta &= \\
z &= \\
\text{Cartesian } & \leftrightarrow \text{ Spherical} \\
x &= \\
y &= \\
z &= \\
\text{Spherical } & \leftrightarrow \text{ Cartesian} \\
\rho &= \\
\theta &= \\
\phi &= \\
\text{For extra credit: Cylindrical } & \leftrightarrow \text{ Spherical and vice versa:} \\
r &= \\
\theta &= \\
z &= \\
\rho &= \\
\theta &= \\
\phi &=
\end{align*}
\]
1.) Find a possible formula for each of the four vector fields pictured below.
Additional information: Each one is of the form \( \vec{F}(x, y) = (ax + by)i + (cx + dy)j \) where each of \( a, b, c, d \) equals one of \(-1, 0, 1\).

For bonus credit: Which of the above vector fields can not be the gradient fields of a smooth function defined everywhere.
No credit without convincing explanation. (Use back side).

2.a) If one zooms in on any point along a smooth parametrized curve passing through a vector field, what will the picture look like after sufficient magnification. Explain in one or two full sentences.

b.) Sketch what one would get, after zooming in on the point \( P \) on the curve in the vector field below. Draw a rough sketch in the box on the right.